2 MOTION IN ONE DIMENSION

Syllabus :

Scalar and vector quantities, distance, speed, velocity, acceleration; graphs of distance-time and speed-time., Equations of uniformly accelerated motion with derivations.

Scope – Examples of scalar and vector quantities only, rest and motion in one dimension, distance and displacement, speed and velocity; acceleration and retardation; distance-time and velocity-time graphs; meaning of slope of the graphs. (Non-uniform acceleration excluded). Equations to be derived : $v = u + at$; $S = ut + \frac{1}{2}at^2$; $S = \frac{1}{2}(u + v)t$; $v^2 = u^2 + 2aS$ (equation for S_n^{th} is not included), Simple numerical problems.

(A) SOME TERMS RELATED TO MOTION

2.1 SCALAR AND VECTOR QUANTITIES

The quantities which we can measure are called the physical quantities. The physical quantities are classified into the following *two* broad categories:

(1) Scalar quantities or scalars, and (2) Vector quantities or vectors.

(1) *Scalar quantities or scalars* : *These are the physical quantities which are expressed only by their magnitude.* For example, if we say that the mass of a body is 5·0 kg, it has a complete meaning and we are completely expressing the mass of the body. Thus, we need the following two parameters to express a scalar quantity completely :

(i) Unit in which the quantity is being measured, and

(ii) The numerical value of the quantity.

Remember that if the scalar is a pure number (like π, e^2, etc.), it will have no unit.

Examples : Mass, length, time, distance, density, volume, speed, temperature, potential (gravitational, magnetic and electric), work, energy, power, pressure, quantity of heat, specific heat, charge, electric power, resistance, density, mechanical advantage, frequency, angle etc.

Scalar quantities can be added, subtracted, multiplied and divided by the simple arithmetic methods. Scalar quantity is symbolically written by its English letter. For example, mass is represented by the letter m, time by t and speed by v.

(2) *Vector quantities or vectors* : *These physical quantities require the magnitude as well as the direction to express them, then only their meaning is complete.* For example, if we say that "displace a particle from a point by 5 m", the first question that will arise, will be "in which direction"? Obviously, by saying that the displacement is 5 m, its meaning is incomplete. But if we say that displace the particle from that point by 5 metre towards east (or in any other direction), it has a complete meaning. Thus, we require the following three meters to express a vector quantity completely :

(i) Unit,

(ii) Numerical value of the quantity and

(iii) Direction.

Examples : Displacement, velocity, acceleration, momentum, force, moment of a force (or torque), impulse, weight, temperature gradient, electric field, magnetic field, dipole moment, *etc*.

The numerical value of a vector quantity alongwith its unit gives us the magnitude of that quantity. It is always positive. The negative sign with a vector quantity implies the reverse (or opposite) direction. Vector quantities follow different algebra for their addition, subtraction and multiplication. A vector quantity is generally written by its English letter bearing an arrow on it or by the bold English letter. For example, velocity is written as $\vec{v}$ or $\boldsymbol{v}$, acceleration by $\vec{a}$ or $\boldsymbol{a}$, force by $\vec{F}$ or $\boldsymbol{F}$. Obviously the forces $\vec{F}$ and $-\vec{F}$ are in opposite directions.

2.2 REST AND MOTION

Every object in the universe is in motion. Everyday we see bodies moving around us e.g. birds flying, cars and buses moving, people walking, insects crawling, animals running etc. Our earth also moves around the sun so every thing on it is in a state of motion. The sun and stars are moving around the centre of their galaxy and the galaxies too are not at rest.

Although nothing is at rest, but we often say that a stone lying on the ground is at rest because the stone does not change its position with respect to us. Similarly, if we are sitting on a railway platform and look at a tree nearby, we say that the tree is at rest because it does not change its position *with respect to us*. But when we see a train leaving the station, we say that the train is in motion because it is continuously changing its position *with respect to us*. Thus,

A body is said to be at rest if it does not change its position with respect to its immediate surroundings, while a body is said to be in motion if it changes its position with respect to its immediate surroundings.

For a moving body, if the distance travelled in a certain time interval is much large as compared to the size of the body, the body can be assumed to be a point particle. In this chapter, we shall study the description of motion of a body assuming it to be a point particle.

One dimensional motion : *When a body moves along a straight line path, its motion is said to be one dimensional motion.* It is also called motion in a straight line or rectilinear motion. For example, the motion of a train on a straight track, a stone falling down vertically, a car moving on a long and straight road etc., are one dimensional (or rectilinear) motions. In such a motion, there is no movement of the body in lateral direction (*i.e.*, no sideways motion).

If a body moves on a plane along a curved path, its motion is two dimensional and if it moves in space, its motion is three dimensional. In this chapter, we shall consider only the one dimensional motion.

Representation of one dimensional motion : The path of one dimensional motion can be represented by a straight line parallel to the *X*-axis if *X*-axis is taken in the direction of motion. Each point on the straight line represents the position of particle at different instants. The position of particle at any instant t is expressed by specifying the x coordinate at that instant. As the particle moves, its x coordinate will change with time t.

Example : The position of a pebble measured from its starting point, falling freely and vertically downwards at different instants is given in the table below :

Time t (in s)	0	1	2	3	4
Position x (in m)	0	5	20	45	80

The motion of the pebble can be represented by choosing a proper scale for x on a straight line along *X*-axis as shown in Fig. 2.1. Here *X*-axis represents the vertically downward direction.

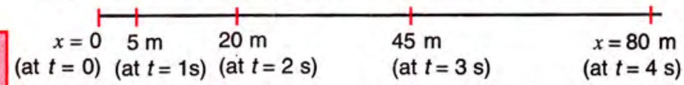

Fig. 2.1 Representation of one-dimensional motion

2.3 DISTANCE AND DISPLACEMENT

Consider a body moving from a point *A* to a point *B* along the path shown in Fig. 2.2. Then total length of path from *A* to *B* is called the *distance* moved by the body, while the length of straight line *AB* in direction from *A* to *B* (shown by the dotted line in Fig. 2.2) is called the *displacement* of the body.

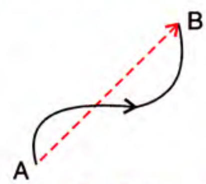

Fig. 2.2 Motion of a body from A to B

Distance

The total length of path through which a body moves, is called the distance travelled by it. The distance travelled by a body depends on the path followed by the body.

It is a **scalar quantity**. It is generally represented by the letter *S*.

Unit : The S.I. unit of distance is metre (m) and C.G.S. unit is centimetre (cm).

Displacement

The shortest distance from the initial to the final position of the body, is the magnitude of displacement and its direction is from the initial position to the final position.

It is a **vector quantity**. It is represented by the symbol $\vec{S}$.

Unit : The S.I. unit of displacement is metre (m) and C.G.S. unit is centimetre (cm).

Representation of displacement : The displacement being a vector, is represented by a straight line with an arrow, using a convenient scale. The tip of arrow on the straight line represents the direction of displacement, while the length of the straight line on proper scale represents its magnitude.

Example: In Fig. 2.3, the vector $\overrightarrow{PQ}$ represents 40 m displacement in east direction with scale 1 cm = 10 m (displacement). Here origin P is the initial position and terminus Q is the final position of the body

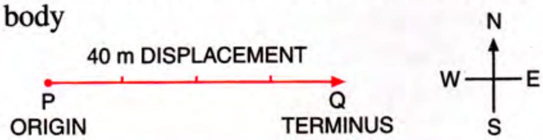

Fig. 2.3 Representation of displacement

Distinction between distance and displacement

(1) ***The magnitude of displacement is either equal to or less than the distance.*** If motion is along a fixed direction, the magnitude of displacement is equal to that of distance, but if motion is along a curve or any zig-zag path, the magnitude of displacement is always less than that of distance. *The magnitude of displacement can never be greater than the distance travelled by the body.*

Examples: **(i)** In Fig. 2.2, the body moves from A to B along a curved path. The distance travelled by the body is equal to the length of the curved path AB, but the displacement of the body is along the straight line AB shown by the dotted arrow. Obviously the magnitude of displacement is less than the distance.

(ii) In Fig. 2.4, a boy travels 4 km towards east and then 3 km towards north. The total distance travelled by the boy is $OA + AB = 4$ km + 3 km = 7 km, but the displacement of the boy is $OB = 5$ km in direction $\overrightarrow{OB}$ *i.e.*, 36·9° due north from east.

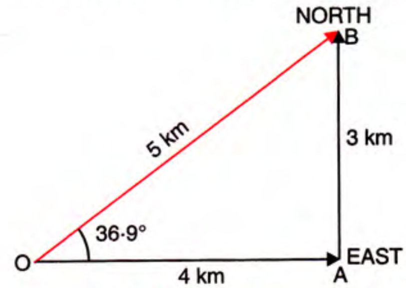

Fig. 2.4 Displacement as a vector

Thus, the magnitude of displacement is the length of the straight line between the final and initial positions.

(2) The distance is the length of path travelled by the body so it is always positive, but displacement is the shortest length in direction from initial position to the final position so it can be positive or negative depending on its direction.

(3) ***The displacement can be zero even if the distance is not zero.*** If a body, after travelling, comes back to its starting point, the displacement is zero but the distance travelled is not zero.

Examples: **(i)** When a body is thrown vertically upwards from a point A on the ground, after some time it comes back to the same point A, then the displacement of the body is zero, but the distance travelled by the body is not zero (it is $2h$ if h is the maximum height attained by the body).

(ii) A body moving in a circular path when reaches its original position after one round, then the displacement at the end of one round is zero, but the distance travelled by it is equal to the circumference of the circular path ($= 2\pi r$ if r is the radius of the circular path).

Distinction between distance and displacement

Distance	Displacement
1. It is the length of the path traversed by the object in a certain time.	1. It is the distance travelled by the object in a specified direction in a certain time (*i.e.*, it is the shortest distance between the final and initial positions).
2. It is a scalar quantity *i.e.*, it has only the magnitude.	2. It is a vector quantity *i.e.*, it has both the magnitude and direction.
3. It depends on the path followed by the object.	3. It does not depend on the path followed by the object.
4. It is always positive.	4. It can be positive or negative depending on its direction.
5. It can be more than or equal to the magnitude of displacement.	5. Its magnitude can be less than or equal to the distance, but can never be greater than the distance.
6. It may not be zero even if displacement is zero, but it can not be zero if displacement is not zero.	6. It is zero if distance is zero, but it can be zero even if distance is not zero.

2.4 SPEED AND VELOCITY

For a moving body, speed is the quantity by which we know how fast the body is moving, while velocity is the quantity by which we know the speed of the body as well as its direction of motion. By speed we do not know the direction of motion of the body.

(1) *Speed*

The speed of a body is the rate of change of distance with time. Numerically it is the distance travelled by the body in 1 s.

It is a **scalar quantity.** It is generally represented by the letter u or v.

If a body travels a distance S in time t, then its speed v is

$$\text{Speed } v = \frac{\text{Distance } S}{\text{Time } t} \qquad ...(2.1)$$

Unit : Unit of speed = $\frac{\text{Unit of distance}}{\text{Unit of time}}$

Since S.I. unit of distance is metre (m) and of time is second (s), so the S.I. unit of speed is metre per second (m s^{-1}) and its C.G.S. unit is centimetre per second (cm s^{-1}).

Uniform speed : *A body is said to be moving with uniform speed if it covers equal distances in equal intervals of time throughout its motion.*

Example : The motion of a ball on a frictionless plane surface is with uniform speed.

Knowing the uniform speed of a body, we can calculate the distance moved by the body in a certain interval of time. If a body moves with a uniform speed v, the distance travelled by it in time t is given as :

$$S = v\,t \qquad ...(2.2)$$

Non-uniform or variable speed : *A body is said to be moving with non-uniform (or variable) speed if it covers unequal distances in equal intervals of time.*

Examples : The motion of a ball on a rough surface, the motion of a car in a crowded street, the motion of a vehicle leaving or approaching a destination etc., are with non-uniform speed.

In case of bodies moving with non-uniform speed, we specify their instantaneous speed and the average speed.

Instantaneous speed : When the speed of a body keeps on changing, its speed at any instant is measured by finding the ratio of the distance travelled in a *very short time interval* to the time interval. This speed is called the instantaneous speed. Thus,

$$\text{Instantaneous speed} = \frac{\text{Distance travelled in a short time interval}}{\text{Time interval}}$$

The speedometer of a vehicle measures the instantaneous speed.

Average speed : The ratio of the total distance travelled by the body to the total time of journey is called its average speed. Thus,

$$\textit{Average speed} = \frac{\textit{Total distance travelled}}{\textit{Total time taken}} \qquad ..(2.3)$$

In case of a body moving with uniform speed, the instantaneous speed and the average speed are equal (same as the uniform speed).

(2) *Velocity*

The velocity of a body is the distance travelled per second by the body in a specified direction.

Thus, *the rate of change of displacement of a body with time is called the velocity.* It is numerically equal to the displacement of the body in 1 s.

It is a **vector quantity** and is represented by the symbol $\vec{u}$ or $\vec{v}$. For velocity, both its magnitude and direction must be specified. Two bodies are said to be moving with same velocities if both of them move with the *same speed in the same direction.* On the other hand, if two bodies move with the *same speed but in different directions or with different speeds in the same direction*, they are said to be moving with different velocities.

Unit : The unit of velocity is same as the unit of speed *i.e.* the S.I. unit of velocity is metre per second (m s^{-1}) and the C.G.S. unit is centimetre per second (cm s^{-1}).

Uniform velocity : *If a body travels equal distances in a particular direction, in equal* *intervals of time, the body is said to be moving with a uniform velocity.*

Example : The rain drops reach on earth's surface falling with uniform velocity*. A body, once started on a frictionless surface, moves with uniform velocity.

If a body moving with a uniform velocity $\vec{v}$, has displacement $\vec{S}$ in a time interval t then by defirition $\vec{v} = \vec{S}/t$.

$\therefore$ Displacement $\boxed{\vec{S} = \vec{v}\ t}$ ...... (2.4)

Non-uniform or variable velocity : The velocity of a body can be variable either due to change in its magnitude or in its direction or in both magnitude and direction. *If a body moves unequal distances in a particular direction in equal intervals of time or it moves equal distances in equal intervals of time, but its direction of motion does not remain the same, then the velocity of the body is said to be variable (or non-uniform).*

Examples : The motion of a freely falling body is with variable velocity because although the direction of motion of the body does not change, but the speed continuously increases. Similarly, the motion of a body in a circular path even with uniform speed is with variable velocity because in a circular path, the direction of motion of the body continuously changes with time. In fact, its velocity changes at a uniform rate. At any instant, its velocity is along the tangent to the circular path at that point. Fig. 2.5 shows the direction of velocity v at different points A, B, C and D of the circular path.

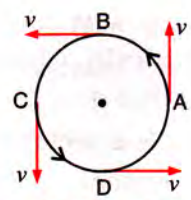

Fig. 2.5 Circular path of constant speed but variable velocity

In case of a body moving with non-uniform velocity, we specify the instantaneous velocity and the average velocity.

Instantaneous velocity : For a body moving with variable velocity, the velocity of the body at any instant is called its instantaneous velocity. It is measured by finding the ratio of the distance travelled in a sufficiently small time interval, to the time interval. It is important to have time interval small enough so that the direction of motion does not change during this interval.

Average velocity : If the velocity of a body moving in a particular direction changes with time, the ratio of displacement to the time taken in entire journey is called its average velocity. Thus,

$$\boxed{\textit{Average velocity} = \frac{\textit{Displacement}}{\textit{Total time taken}}} \qquad ...(2.5)$$

Distinction between speed and velocity

(1) The speed is a scalar quantity, while velocity is a vector quantity. The speed of a body at a given time tells us how fast the body is moving at that time. The same information is also obtained by its velocity, but the velocity also tells us the direction in which the body is moving.

(2) For the motion in a straight line, the magnitude of velocity is its speed. The speed is always positive, but velocity is given positive or negative sign depending upon its direction of motion.

(3) *The average velocity of a body can be zero, even if its average speed is not zero.* **Examples : (i)** If a body starts its motion from a point and comes back to the same point after a certain time, the displacement is zero, so the average velocity is also zero, but the total distance travelled is not zero and therefore, the average speed is not zero.

(ii) If a body moves in a circular path and covers equal distances in equal intervals of time, the speed is uniform, but due to continuous change in its direction of motion, its velocity is variable. The instantaneous velocity and instantaneous speed are not zero. The displacement for one round is zero and therefore, the average velocity is also zero, but the average speed is $2\pi r/T$ if r is the radius of path and T is the time taken in one round.

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* *Initially as the rain drop starts falling, first its velocity increases due to force of gravity, but very soon, due to viscosity (or friction) and upthrust of air, the viscous force and upthrust balances the force of gravity on the rain drop with the result that the net force on the drop becomes zero. Then the drop falls down with a uniform velocity called the terminal velocity.*

Distinction between speed and velocity

Speed	Velocity
1. The distance travelled per second by a moving object is called its speed.	1. The distance travelled per second by a moving object in a particular direction is called its velocity.
2. It is a scalar quantity. The speed does not tell us the direction of motion.	2. It is a vector quantity. The velocity tells us the speed as well as the direction of motion.
3. The speed is always positive since direction is not taken into consideration.	3. The velocity can be positive or negative depending upon the direction of motion.
4. After one round in a circular path, the average speed is not zero.	4. After completing each round in a circular path, the average velocity is zero.

2.5 ACCELERATION AND RETARDATION

Generally, bodies do not move with uniform velocities. The velocity of a body changes either in magnitude or in direction or both in magnitude as well as in direction. For example, the motion of a vehicle in a busy market, or while leaving or approaching a destination is with variable velocity. The motion of a planet or satellite in circular path is also with variable velocity.

Now we consider the motion only in a straight line path. Here there is no change in direction of motion and the change in velocity is only due to change in speed. In such a case, if the velocity of body increases with time, the motion is said to be *accelerated,* while if the velocity of body decreases with time, the motion is said to be *decelerated* (or *retarded*). Thus *retardation is the negative acceleration**.

Acceleration

Acceleration is the rate of change of velocity with time.*

Thus, acceleration is numerically equal to the change in velocity in 1 s. *i.e.*,

$$\text{Acceleration} = \frac{\text{Change in velocity}}{\text{Time interval}} \quad ..(2.6)$$

Unit : Unit of acceleration $= \dfrac{\text{Unit of velocity}}{\text{Unit of time}}$

The S.I. unit of velocity is metre per second and of time is second.

$\therefore$ S.I. unit of acceleration is $\dfrac{\text{metre per second}}{\text{second}}$

= metre per second square or m s^{-2}.

* *Acceleration is the increase in velocity per second, while retardation is the decrease in velocity per second.*

The C.G.S. unit of acceleration is cm s^{-2}.

Relation for acceleration : Let a body be moving in a straight line in one direction with an initial velocity u. Its velocity changes in a short time interval t and the final velocity becomes v after time t. Then change in velocity = $(v - u)$ and time taken = t.

$$\left.\begin{aligned} \therefore \quad \text{Acceleration } a &= \frac{(v-u)}{t} \\ \text{or} \quad v &= u + a\,t \end{aligned}\right\} \quad ....(2.7)$$

If $v > u$, then a is positive, thus a is the acceleration. But if $v < u$, then a is negative, and a is the retardation.

Examples: (1) Suppose a car initially at rest, on starting acquires a velocity 20 m s^{-1} in 10 s. The change in its velocity is 20 m s^{-1} – 0 m s^{-1} = 20 m s^{-1}. This change has been brought about in 10 s. Therefore, the acceleration of car is

$$a = \frac{20\,\text{m s}^{-1} - 0\,\text{m s}^{-1}}{10\ \text{s}} = \frac{20\,\text{m s}^{-1}}{10\,\text{s}} = 2\ \text{m s}^{-2}.$$

(2) If a car initially moving with a velocity 25 m s^{-1} is brought to rest in 5 s by applying the brakes, then acceleration of the car is

$$a = \frac{0\,\text{m s}^{-1} - 25\,\text{m s}^{-1}}{5\ \text{s}} = \frac{-25\,\text{m s}^{-1}}{5\,\text{s}} = -5\ \text{m s}^{-2}$$

or retardation = 5 m s^{-2} (since negative acceleration is called the retardation).

Acceleration is a **vector quantity**. It is represented by the symbol $\vec{a}$. The direction of acceleration is the direction of change in velocity. For the motion in a straight line, the acceleration is in direction of motion of the body.

It may be mentioned here that *the acceleration of a body does not determine its direction of motion, while the velocity determines its direction of motion.* The positive or negative sign of acceleration tells us whether the velocity is increasing or decreasing with time, whereas the positive or negative sign of velocity tells its direction of motion.

Uniform acceleration : *The acceleration is said to be uniform (or constant) when equal changes in velocity take place in equal intervals of time.* The motion of a body under gravity (e.g. free fall of a body) is an example of uniformly accelerated motion.

Variable acceleration : *If change in velocity is not same in the same intervals of time, the acceleration is said to be variable.* The motion of a vehicle on a crowded (or hilly) road is with variable acceleration.

Acceleration due to gravity : *When a body falls freely under gravity, the acceleration produced in the body due to earth's gravitational attraction is called the acceleration due to gravity.* It is generally denoted by the letter g. When a body falls down, its velocity increases with time, so the acceleration is $+ g$, while if the body moves vertically upwards, its velocity decreases with time, so the acceleration is $- g$ (or the retardation is g).

The average value of g is $9{\cdot}8\ m\ s^{-2}$ (or nearly $10\ m\ s^{-2}$). Actually it varies from place to place*. Thus if a body falls freely under gravity, its velocity increases at a rate of $9{\cdot}8\ m\ s^{-2}$, *i.e.*, starting from rest, after 1 s the velocity will be $9{\cdot}8\ m\ s^{-1}$, after 2 s the velocity will be $2 \times 9{\cdot}8 = 19{\cdot}6\ m\ s^{-1}$; after 3 s, the velocity with be $3 \times 9{\cdot}8 = 29{\cdot}4\ m\ s^{-1}$ and so on. Similarly, if a body is projected vertically upwards, its velocity decreases at a rate of $9{\cdot}8\ m\ s^{-2}$, *i.e.*, if a body is projected vertically upwards with an initial velocity of $49\ m\ s^{-1}$, after 1 s, its velocity will become $39{\cdot}2\ m\ s^{-1}$; after 2 s, its velocity will become $29{\cdot}4\ m\ s^{-1}$; after 3 s, its velocity will become $19{\cdot}6\ m\ s^{-1}$ and so on.

Note : The value of g does not depend on the mass of the body. Hence if two bodies of different masses are simultaneously dropped from a height, both will reach the ground simultaneously in vacuum because then there is no effect of friction and buoyancy due to air.

* *On the earth surface, g is maximum at the poles and minimum at the equator. The value of g decreases with altitude and also with depth from the earth's surface.*

EXAMPLES

1. Select the scalars and vectors from the following :
Velocity, distance, acceleration, work, mass, retardation.

Scalars : distance, work, mass.

Vectors : velocity, acceleration, retardation.

2. Express the speed 36 km h^{-1} in m s^{-1}.

$$36\ km\ h^{-1} = \frac{36\,km}{1\,h} = \frac{36 \times 1000\,m}{60 \times 60\,s} = \mathbf{10\ m\ s^{-1}}$$

3. Find the distance travelled by a body in 5 minutes if it travels with a uniform speed of 20 m s^{-1}.

Given, $u = 20\ m\ s^{-1}$, $t = 5\ min = 5 \times 60\ s = 300\ s$

Distance travelled S = speed u × time t
= $20\ m\ s^{-1} \times 300\ s$
= 6000 m = **6 km.**

4. A train moving with uniform speed covers a distance of 120 m in 2 s. Calculate : (i) the speed of the train, (ii) the time it will take to cover 240 m.

Given, $S = 120$ m, $t = 2$ s

(i) Speed of the train = $\dfrac{\text{Distance travelled}}{\text{Time taken}}$

or $u = \dfrac{120\ m}{2\ s} = \mathbf{60\ m\ s^{-1}}$

(ii) Time taken to cover 240 m distance

$$t = \frac{\text{distance}}{\text{speed}} = \frac{240\ m}{60\ m\ s^{-1}} = \mathbf{4\ s}$$

5. A body rises vertically up to a height of 125 m in 5 s and then comes back at the point of projection. Find : (i) the total distance travelled, (ii) the displacement, (iii) the average speed and (iv) the average velocity of the body.

Given, $S = 125$ m, $t = 5$ s

(i) Total distance travelled $= S + S = 2S$

$= 2 \times 125$ m

$= \mathbf{250\ m}$

(ii) Displacement = **0** (since final position is same as initial position).

(iii) Average speed $= \dfrac{\text{Total distance travelled}}{\text{Total time of journey}}$

$$= \frac{2S}{2t} = \frac{2 \times 125\ \text{m}}{2 \times 5\ \text{s}}$$

$$= \frac{250\ \text{m}}{10\ \text{s}} = \mathbf{25\ m\ s^{-1}}$$

(iv) Average velocity = **0** (since displacement is zero).

6. A train first travels for 30 min with a velocity 30 km h^{-1} and then for 40 min with a velocity 40 km h^{-1} in the same direction, Calculate : (i) the total distance travelled, (ii) the average velocity of the train.

Given, $t_1 = 30\ \text{min} = \frac{1}{2}\ \text{h}$, $v_1 = 30\ \text{km h}^{-1}$,

$t_2 = 40\ \text{min} = \frac{2}{3}\ \text{h}$, $v_2 = 40\ \text{km h}^{-1}$.

(i) Distance travelled = velocity × time

$\therefore\ S_1 = v_1 \times t_1 = 30 \times \frac{1}{2} = 15\ \text{km}$

$S_2 = v_2 \times t_2 = 40 \times \frac{2}{3} = \frac{80}{3}\ \text{km}$

Total distance travelled $S = S_1 + S_2$

$$= 15 + \frac{80}{3} = \frac{125}{3}\ \text{km} = \mathbf{41{\cdot}67\ km}$$

(ii) Total time of journey $t = t_1 + t_2$

$$= \frac{1}{2} + \frac{2}{3} = \frac{7}{6}\ \text{h}$$

$\therefore$ Average velocity $= \dfrac{\text{Total distance travelled } S}{\text{Total time of journey } t}$

$$= \frac{41 \cdot 67\ \text{km}}{(7/6)\ \text{h}} = \mathbf{35{\cdot}71\ km\ h^{-1}}.$$

7. A car travels a distance 50 km with a velocity 25 km h^{-1} and then 60 km with a velocity 20 km h^{-1} in the same direction. Calculate : (i) the total time of journey and (ii) the average velocity of the car.

Given, $S_1 = 50\ \text{km}$, $v_1 = 25\ \text{km h}^{-1}$,

$S_2 = 60\ \text{km}$, $v_2 = 20\ \text{km h}^{-1}$.

(i) Time of journey $t = \dfrac{\text{Distance } S}{\text{velocity } v}$

$$\therefore\quad t_1 = \frac{S_1}{v_1} = \frac{50\ \text{km}}{25\ \text{km h}^{-1}} = 2\ \text{h}$$

$$t_2 = \frac{S_2}{v_2} = \frac{60\ \text{km}}{20\ \text{km h}^{-1}} = 3\ \text{h}$$

Total time of journey $t = t_1 + t_2 = 2\ \text{h} + 3\ \text{h} = \mathbf{5\ h}$

(ii) Total distance travelled $S = S_1 + S_2$

$= 50\ \text{km} + 60\ \text{km} = 110\ \text{km}$

Average velocity $= \dfrac{\text{Total distance travelled } S}{\text{Total time of journey } t}$

$$= \frac{110\ \text{km}}{5\ \text{h}} = \mathbf{22\ km\ h^{-1}}.$$

8. The velocity of an object increases at a constant rate from 20 m s^{-1} to 50 m s^{-1} in 10 s. Find the acceleration.

Given, $u = 20\ \text{m s}^{-1}$, $v = 50\ \text{m s}^{-1}$, $t = 10$ s

$$\text{Acceleration } a = \frac{v-u}{t} = \frac{50\ \text{m s}^{-1} - 20\ \text{m s}^{-1}}{10\ \text{s}}$$

or $$a = \frac{30\ \text{m s}^{-1}}{10\ \text{s}} = \mathbf{3\ m\ s^{-2}}.$$

9. A pebble thrown vertically upwards with an initial velocity 50 m s^{-1} comes to a stop in 5 s. Find the retardation.

Given, $u = 50\ \text{m s}^{-1}$, $v = 0$, $t = 5$ s

$$\text{Acceleration } a = \frac{v-u}{t} = \frac{0 - 50\ \text{m s}^{-1}}{5\ \text{s}}$$

$$= -10\ \text{m s}^{-2}$$

Hence, retardation = $\mathbf{10\ m\ s^{-2}}$.

10. The table below shows the distance in cm, travelled by the objects *A*, *B* and *C* during each second.

Time	Distance (in cm) covered in each second by *A*, *B* and *C*		
	Object *A*	Object *B*	Object *C*
1st second	20	20	20
2nd second	20	36	60
3rd second	20	24	100
4th second	20	30	140
5th second	20	48	180

(i) Which object is moving with constant speed ? Give a reason for your answer.

(ii) Which object is moving with a constant acceleration ? Give a reason.

(iii) Which object is moving with irregular acceleration ?

(i) The object *A* is moving with constant speed. The reason is that it covers equal distance (= 20 cm) in each second.

(ii) The object *C* is moving with a constant acceleration. The reason is that for the object *C*, the distance covered in each second (*i.e.*, velocity) increases by the same amount (= 40 m s^{-1}) in each interval of one second.

(iii) The object ***B*** is moving with irregular acceleration. The reason is that the change in velocity is not same in each interval of one second.

EXERCISE 2(A)

1. Differentiate between the scalar and vector quantities, giving *two* examples of each.
2. State whether the following quantity is a scalar or vector ?
   (a) pressure (b) force
   (c) momentum (d) energy
   (e) weight (f) speed.
   Ans. (a) scalar (b) vector (c) vector (d) scalar (e) vector (f) scalar.
3. When is a body said to be at rest ?
4. When is a body said to be in motion ?
5. What do you mean by motion in one direction ?
6. Define displacement. State its unit.
7. Differentiate between distance and displacement.
8. Can displacement be zero even if distance is not zero ? Give *one* example to explain your answer.
9. When is the magnitude of displacement equal to the distance ?
   Ans. When the motion is in a fixed direction
10. Define velocity. State its unit.
11. Define speed. What is its S.I. unit ?
12. Distinguish between speed and velocity.
13. Which of the quantity speed or velocity gives the direction of motion of body ? **Ans.** Velocity
14. When is the instantaneous speed same as the average speed ?
15. Distinguish between the uniform velocity and the variable velocity.
16. Distinguish between the average speed and the average velocity.
17. Give an example of the motion of a body moving with a constant speed, but with a variable velocity. Draw a diagram to represent such a motion.
18. Give an example of motion in which average speed is not zero, but the average velocity is zero.
19. Define acceleration. State its S.I. unit.
20. Distinguish between acceleration and retardation.
21. Differentiate between uniform acceleration and variable acceleration.
22. What is meant by the term retardation ? Name its S.I. unit.
23. Which of the quantity, velocity or acceleration determines the direction of motion ? **Ans.** Velocity
24. Give *one* example of each type of the following motion :
   (a) uniform velocity (b) variable velocity
   (c) variable acceleration (d) uniform retardation.
25. The diagram (Fig. 2.6) below shows the pattern of the oil dripping on the road, at a constant rate from a moving car. What informations do you get from it about the motion of car ?

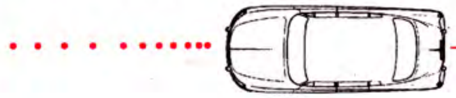


Fig. 2.6

   Ans. Initially it is moving with a constant speed and then it slows down.
26. Define the term acceleration due to gravity. State its average value.
27. 'The value of *g* remains same at all places on the earth surface'. Is this statement true ? Give reason for your answer.
   Ans. No. The value of *g* is maximum at poles and minimum at equator on the earth surface.
28. If a stone and a pencil are dropped simultaneously in vacuum from the top of a tower, which of the two will reach the ground first ? Give reason.
   Ans. Both will reach the ground simultaneously, since acceleration due to gravity is same (= *g*) on both.

Multiple choice type :

1. The vector quantity is :
   (a) work (b) pressure
   (c) distance (d) velocity
   Ans. (d) velocity
2. The S.I. unit of velocity is :
   (a) km h^{-1} (b) m min^{-1}
   (c) km min^{-1} (d) m s^{-1} **Ans.** (d) m s^{-1}
3. The unit of retardation is :
   (a) m s^{-1} (b) m s^{-2}
   (c) m (d) m s^2 **Ans.** (b) m s^{-2}
4. A body when projected up with an initial velocity *u* goes to a height *h* in time *t* and then comes back at the point of projection. The correct statement is :
   (a) the average velocity is $2h/t$
   (b) the acceleration is zero

(c) the final velocity on reaching the point of projection is $2u$.

(d) the displacement is zero.

Ans. (d) the displacement is zero.

5. 18 km h^{-1} is equal to :

(a) 10 m s^{-1} (b) 5 m s^{-1}

(c) 18 m s^{-1} (d) 1·8 m s^{-1} **Ans.** (b) 5 m s^{-1}

Numericals :

1. The speed of a car is 72 km h^{-1}. Express it in m s^{-1}. **Ans.** 20 m s^{-1}
2. Express 15 m s^{-1} in km h^{-1}. **Ans.** 54 km h^{-1}
3. Express each of the following in m s^{-1} :
   (a) 1 km h^{-1} (b) 18 km min^{-1}
   Ans. (a) 0·278 m s^{-1} (b) 300 m s^{-1}
4. Arrange the following speeds in increasing order : 10 m s^{-1}, 1 km min^{-1}, 18 km h^{-1}.
   Ans. 18 km h^{-1}, 10 m s^{-1}, 1 km min^{-1}
5. A train takes 3 h to travel from Agra to Delhi with a uniform speed of 65 km h^{-1}. Find the distance between the two cities. **Ans.** 195 km
6. A car travels first 30 km with a uniform speed of 60 km h^{-1} and then next 30 km with a uniform speed of 40 km h^{-1}. Calculate : (i) the total time of journey, (ii) the average speed of the car.
   Ans. (i) 75 min, (ii) 48 km h^{-1}.
7. A train takes 2 h to reach station B from station A, and then 3 h to return from station B to station A. The distance between the two stations is 200 km. Find : (i) the average speed, (ii) the average velocity of the train. **Ans.** (i) 80 km h^{-1}, (ii) zero.
8. A car moving on a straight path covers a distance of 1 km due east in 100 s. What is (i) the speed and (ii) the velocity, of car ?
   Ans. (i) 10 m s^{-1}, (ii) 10 m s^{-1} due east.
9. A body starts from rest and acquires a velocity 10 m s^{-1} in 2 s. Find the acceleration.
   Ans. 5 m s^{-2}
10. A car starting from rest acquires a velocity 180 m s^{-1} in 0·05 h. Find the acceleration.
    Ans. 1 m s^{-2}
11. A body is moving vertically upwards. Its velocity changes at a constant rate from 50 m s^{-1} to 20 m s^{-1} in 3 s. What is its acceleration ?

Ans. -10 m s^{-2}. The negative sign shows that the velocity decreases with time, so retardation is 10 m s^{-2}.

12. A toy car initially moving with a uniform velocity of 18 km h^{-1} comes to a stop in 2 s. Find the retardation of the car in S.I. units. **Ans.** 2·5 m s^{-2}
13. A car accelerates at a rate of 5 m s^{-2}. Find the increase in its velocity in 2 s. **Ans.** 10 m s^{-1}
14. A car is moving with a velocity 20 m s^{-1}. The brakes are applied to retard it at a rate of 2 m s^{-2}. What will be the velocity after 5 s of applying the brakes ?. **Ans.** 10 m s^{-1}
15. A bicycle initially moving with a velocity 5·0 m s^{-1} accelerates for 5 s at a rate of 2 m s^{-2}. What will be its final velocity ? **Ans.** 15·0 m s^{-1}
16. A car is moving in a straight line with speed 18 km h^{-1}. It is stopped in 5 s by applying the brakes. Find : (i) the speed of car in m s^{-1}, (ii) the retardation and (iii) the speed of car after 2 s of applying the brakes.
    Ans. (i) 5 m s^{-1}, (ii) 1 m s^{-2}, (iii) 3 m s^{-1}

(B) GRAPHICAL REPRESENTATION OF LINEAR MOTION

If a body moves in a straight line path, its motion is in one dimension and is called the linear or rectilinear motion. A linear motion can be well studied with the help of the following graphs :

(i) Displacement-time graph,

(ii) Velocity-time graph, and

(iii) Acceleration-time graph.

Note : For motion in one direction in a straight line since the direction of motion does not change, so the displacement-time graph and the distance-time graph are same. Similarly the velocity-time graph and the speed-time graph are also same. They differ in two and three dimensional motions.

2.6 DISPLACEMENT-TIME GRAPH

In the displacement-time graph, the time is taken on X-axis and the displacement of body is taken on Y-axis. From this graph, we can determine the velocity of the body.

Since, velocity is the ratio of displacement and time, therefore *the slope of displacement-time graph gives the velocity*. If the slope is positive, it implies that the body is moving away from the starting (or reference) point, but if the slope is negative, the body is returning towards the starting (or reference) point.

Let us consider the following situations:

Case (1) : If the position of a body does not change with time, the body is said to be *stationary* and the displacement as measured from the origin at all instant is same as that at $t = 0$, so the displacement-time graph is a straight line parallel to the time axis as shown in Fig. 2.7. In Fig. 2.7, OP is the distance of the body from the origin O and it remains same at each instant.

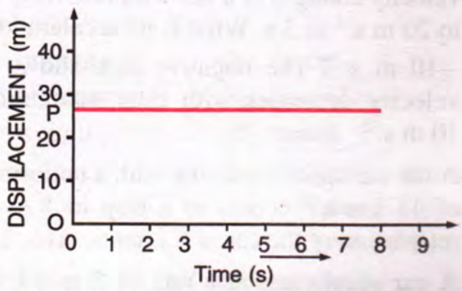


Fig. 2.7 Displacement-time graph for a stationary object

Case (2) : If a body is moving with uniform velocity, its displacement increases by the same amount in each second and so the displacement-time graph is a straight line inclined to the time axis. The velocity of body can be obtained by finding the slope of the straight line.

Example (1) : A car is moving on a *straight path in a given direction* with a uniform speed. The following table represents its displacement (*i.e.*, distance from the starting point) at different instants.

Time in second (s)	0	1	2	3	4	5
Displacement in metre (m)	0	10	20	30	40	50

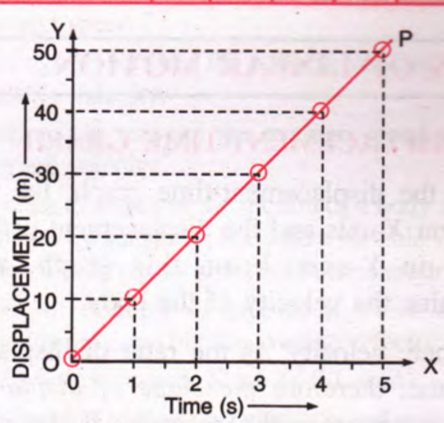


Fig. 2.8 Displacement-time graph with a uniform velocity

The displacement-time graph is a straight line OP inclined to the time axis (Fig. 2.8). It shows the linear relationship between the displacement and time (*i.e.*, the displacement of car is directly proportional to the time of travel or the car travels equal distance in equal intervals of time in a certain direction). Thus

> *For a body moving with uniform velocity, the displacement is directly proportional to the time (i.e., $S \propto t$)*

The velocity of car can be obtained by finding the slope of the straight line OP. For this, take any two points A and B on the line OP as shown in Fig. 2.9 and draw perpendiculars AD and BE on the Y-axis and AF and BG on the X-axis from these points.

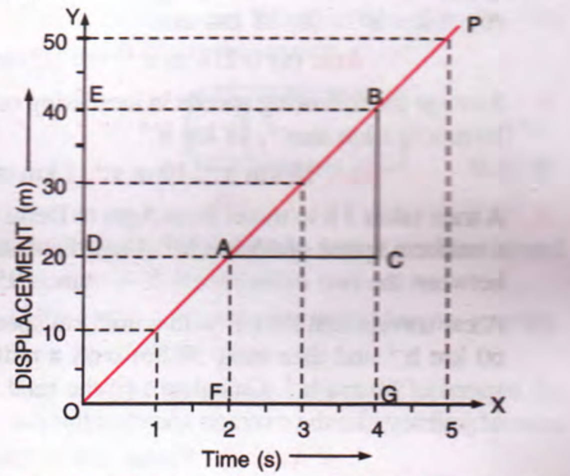


Fig. 2.9 To find velocity from displacement-time graph

From graph in Fig. 2.9, velocity of car

$= $ slope of the line OP

$$= \frac{BC}{CA} = \frac{ED}{GF} = \frac{(40-20)\text{ m}}{(4-2)\text{ s}} = \frac{20\text{ m}}{2\text{ s}} = 10\text{ m s}^{-1}$$

Obviously, *larger the slope (i.e., more inclined is the straight line), higher is the velocity.*

Example (2) : Fig. 2.10 represents the displacement-time graph of a ball which while moving on a perfectly smooth floor hits a wall at $t = 6$ s and then comes back along the same line. The displacement (*i.e.*, distance of ball from the starting point) at different instants of time is given in the table below.

Time (in second)	0	2	4	6	8	10	12	14
Displacement (in metre)	0	20	40	60	45	30	15	0

In Fig. 2.10,

Slope of the line $OA = \dfrac{60\text{ m} - 0}{6\text{ s} - 0} = 10\text{ m s}^{-1}$

and slope of the line $AB = \dfrac{0 - 60\text{ m}}{14\text{ s} - 6\text{ s}} = -\dfrac{60\text{ m}}{8\text{ s}}$

$= -7{\cdot}5\text{ m s}^{-1}$

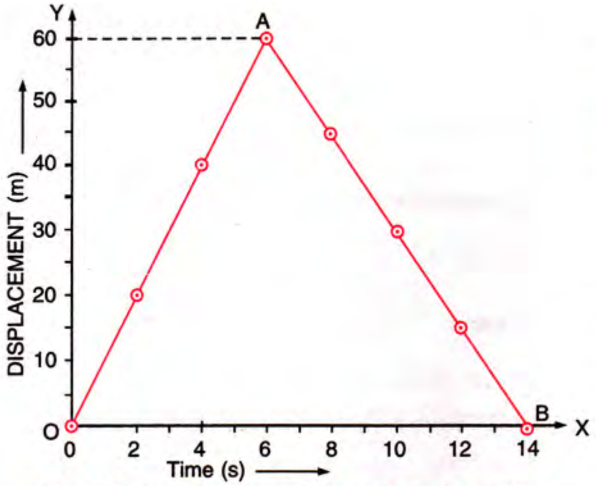


Fig. 2.10 Displacement-time graph with positive and negative slopes

Thus the slope of the line OA is positive ($= 10 \text{ m s}^{-1}$) which represents the uniform motion of ball with velocity 10 m s^{-1} towards the wall (*i.e.*, away from the starting point), while the slope of the line AB is negative ($= -7{\cdot}5 \text{ m s}^{-1}$) which represents the uniform motion of ball with velocity $7{\cdot}5 \text{ m s}^{-1}$ towards the starting point after hitting the wall.

Note : The displacement–time graph can never be a straight line, parallel to the displacement axis because such a line would mean that the distance covered by the body in a certain direction increases without any increase in time (*i.e.*, the velocity of the body is infinite) which is impossible.

Case (3) : If a body moves with varying speed in a fixed direction *i.e.*, with variable velocity, the displacement-time graph is not a straight line, but it is a curve. *The velocity at any instant can then be obtained by finding the slope (or the gradient) of the tangent drawn on the curve at that instant of time.*

Example : Fig. 2.11 represents the displacement-time graph of a body for which displacement (*i.e.*, distance from the starting point) at different instants is given in the table below.

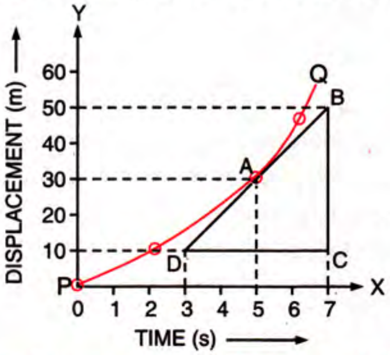


Fig. 2.11 Displacement-time graph with a variable velocity

Time (in second)	0	2	5	6
Displacement (in metre)	0	10	30	47

The displacement-time graph is a curve PQ.

The velocity of body at time $t = 5$ s (or when the displacement $S = 30$ m) is obtained by finding the slope of the tangent BD to the curve drawn at the point A corresponding to $t = 5$ s or $S = 30$ m. Slope of the tangent BD is

$$\text{Slope} = \frac{BC}{CD} = \frac{(50-10)\text{ m}}{(7-3)\text{ s}} = \frac{40 \text{ m}}{4 \text{ s}} = 10 \text{ m s}^{-1}$$

i.e., the velocity of body at $t = 5$ s or $S = 30$ m is 10 m s^{-1}.

Note that in this case, the slope of the curve is different at different points, so the velocity is different at different instants.

Conclusions :

(i) (a) If the displacement-time graph of an object, is a straight line parallel to the time axis, the object is stationary. (b) If the graph is a straight line inclined to the time axis, the motion is with uniform velocity. (c) If the graph is a curve, the motion is with non-uniform velocity.

(ii) In the displacement-time graph, the slope of the straight line (or the tangent to the curve at an instant) gives the velocity of the object at that instant. (a) If the slope is positive, it represents the motion away from the origin (or reference point). (b) If the slope is negative, it represents the motion towards the origin.

(iii) Knowing the velocity of the object at different instants from the displacement-time graph, the velocity-time graph can be drawn.

2.7 VELOCITY-TIME GRAPH

In the velocity-time graph, time is taken on X-axis and the velocity is taken on Y-axis.

Since velocity is a vector quantity, the positive velocity means that the body is moving in a certain direction away from its initial position and the negative velocity means that the body is moving in the opposite direction (*i.e.*, towards the initial position).

From the velocity-time graph, we can determine (a) the displacement of the body in a certain time interval and (b) the acceleration of the body at any instant.

(a) ***Determination of displacement from the velocity-time graph :*** Since, velocity × time = displacement, *the area enclosed between the velocity-time sketch and X-axis (i.e., the time axis) gives the displacement of the body.*

The area enclosed above the time axis represents the positive displacement *i.e.,* the distance travelled away from the starting point, while the area enclosed below the time axis represents the negative displacement *i.e.,* the distance travelled towards the starting point. The total displacement is obtained by adding them numerically with proper sign. But *the total distance travelled by the body is their arithmetic sum* (without sign).

Example : Consider the velocity-time graph of a body in motion as shown in Fig. 2.12.

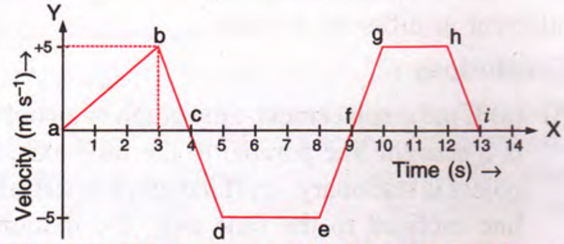


Fig. 2.12 Velocity-time graph of a body in motion

In Fig. 2.12, area of Δ *abc*

$$= \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{1}{2} \times 4\ \text{s} \times 5\ \text{m s}^{-1} = 10\ \text{m},$$

area of trapezium *cdef*

$$= \frac{1}{2} \times (\text{sum of parallel sides}) \times \text{height}$$

$$= \frac{1}{2} \times (5+3)\ \text{s} \times 5\ \text{m s}^{-1} = 20\ \text{m},$$

and area of trapezium *fghi*

$$= \frac{1}{2} \times (4+2)\ \text{s} \times 5\ \text{m s}^{-1} = 15\ \text{m}$$

Then the displacement of the body = area of Δ *abc* – area of trapezium *cdef* + area of trapezium *fghi* = 10 m – 20 m + 15 m = 5 m, but the total distance travelled by body = area of Δ *abc* + area of trapezium *cdef* + area of trapezium *fghi* = 10 m + 20 m + 15 m = 45 m.

(b) ***Determination of acceleration from the velocity-time graph :*** Since acceleration is equal to the ratio of change in velocity and time taken, therefore *the slope (or gradient) of the velocity-time sketch gives the acceleration.*

Example : In Fig. 2.12, for the part *ab* of the motion

$$\text{Slope} = \frac{\text{Change in velocity}}{\text{Change in time}} = \frac{(5-0)\ \text{m s}^{-1}}{(3-0)\ \text{s}}$$

$$\therefore \text{Acceleration} = \frac{5}{3} = 1{\cdot}67\ \text{m s}^{-2}$$

In part *bd*,

$$\text{Slope} = \frac{[(-5)-5]\ \text{m s}^{-1}}{(5-3)\ \text{s}} = -5\ \text{m s}^{-2}$$

$$\therefore \text{Acceleration} = -5\ \text{m s}^{-2}$$

In part *de*, slope = 0, $\therefore$ Acceleration = 0

Since the slope is positive in part *ab*, it is the accelerated motion; the slope is negative in part *bd*, the motion is decelerated or retarded and in part *de* the slope is zero, so the motion is with constant velocity.

Now we can consider the following cases :

Case (1) : If a body is in motion with *uniform velocity* (*i.e.*, velocity remains constant with time), the velocity-time graph is a straight line parallel to the time axis.

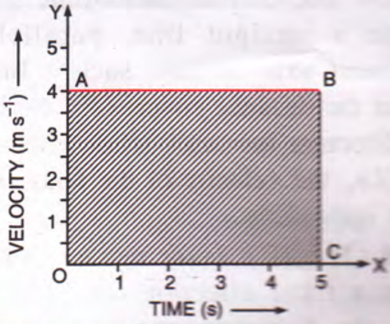


Fig. 2.13. Velocity-time graph for uniform velocity

Example : In Fig. 2.13, a straight line *AB* represents the velocity-time graph of a body moving with a uniform velocity 4 m s^{-1} for 5 s. The slope of the straight line *AB* is zero, therefore, its acceleration is zero.

Displacement in 5 second = area of the rectangle *OABC* = *OC* × *OA* = 5 s × 4 m s^{-1} = 20 m.

Case (2) : (a) If the body is in motion with uniform acceleration (*i.e.*, equal changes in velocity take place in equal intervals of time), the velocity-time graph is a straight line inclined to the time axis. *The slope of the line gives the acceleration.*

Example : The table below represents the velocity of a body at different instants, starting from rest.

Time (in s)	0	1	2	3	4	5	6	7	8
Velocity (in m s^{-1})	0	10	20	30	40	50	60	70	80

Obviously, the velocity is increasing by an equal amount in each second *i.e.,* the body is moving with uniform acceleration. The velocity-time graph is a straight line *OP* inclined to the time axis as shown in Fig. 2.14.

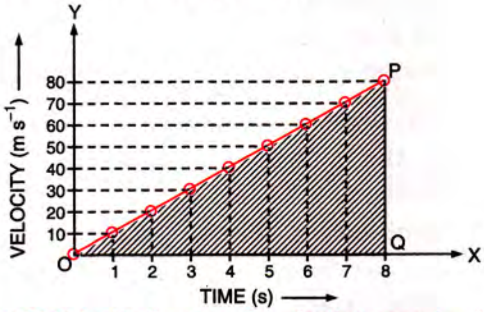


Fig. 2.14 Velocity-time graph; velocity increasing by equal amount in each second

Distance travelled by the body in 8 second

$$S = \text{area of triangle } OPQ$$

$$= \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{1}{2} \times OQ \times QP$$

$$= \frac{1}{2} \times 8 \text{ s} \times 80 \text{ m s}^{-1} = 320 \text{ m}.$$

Acceleration of the body = Slope of the line *OP*

$$= \frac{PQ}{QO} = \frac{(80-0) \text{ m s}^{-1}}{(8-0) \text{ s}}$$

$$= \frac{80 \text{ m s}^{-1}}{8 \text{ s}} = 10 \text{ m s}^{-2}$$

(b) If the motion is with uniform retardation (*i.e.,* its velocity decreases by an equal amount in each second), the velocity-time graph will be a straight line inclined to the time axis with a negative slope.

Example : In Fig. 2.15, the straight line *AB* represents the velocity-time graph for a body initially moving with a velocity 40 m s^{-1} which comes to a stop in 4 s with uniform retardation.

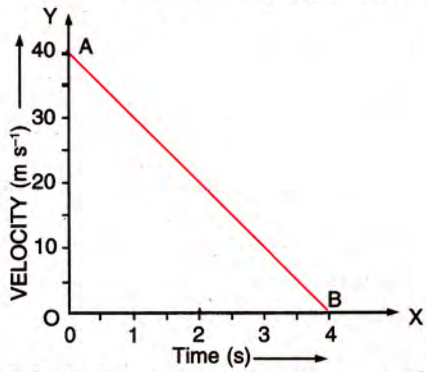


Fig. 2.15 Velocity-time graph with uniform retardation

Distance travelled by the body in 4 s

$$= \text{Area of triangle } AOB$$

$$= \frac{1}{2} OB \times OA$$

$$= \frac{1}{2} \times 4 \text{ s} \times 40 \text{ m s}^{-1} = 80 \text{ m}$$

Retardation of the body

$$= -\text{ Slope of the line } AB$$

$$= -\frac{OA}{BO} = -\frac{(0-40) \text{ m s}^{-1}}{(4-0) \text{ s}}$$

$$= \frac{40}{4} \text{ m s}^{-2} = 10 \text{ m s}^{-2}.$$

Obviously, *on velocity-time graph, larger the slope (i.e., more inclined is the straight line), higher is the acceleration or retardation.*

(c) The velocity-time graph can never be a straight line parallel to the velocity axis because such a line would mean that the velocity increases without any increase in time (*i.e.,* acceleration is infinite) which is impossible.

(d) If the body initially is moving with some velocity and then it accelerates, the velocity-time sketch for the accelerated motion will start from the point on the velocity axis corresponding to the initial velocity of the body.

Example : In Fig. 2.16, straight line *AB* represents the velocity-time graph of a car initially moving with velocity 10 m s^{-1} and then with uniform acceleration. Its velocity at different instants is as given in the following table :

Time (in second)	0	1	2	3	4	5
Velocity (in m s^{-1})	10	15	20	25	30	35

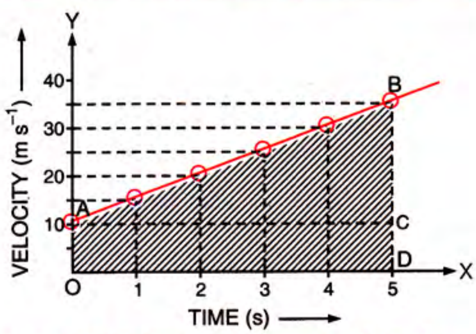


Fig. 2.16 Velocity-time graph when the body is initially not at rest

(i) Displacement of the car in 5 second

$= \text{Area of trapezium } ABDO$

$= \frac{1}{2}(OA + DB) \times OD$

$= \frac{1}{2} \times (10 + 35) \times 5$

$= \frac{1}{2} \times 45 \times 5 = 112{\cdot}5 \text{ m}$

(ii) Acceleration of the car = slope of the line AB

$= \frac{BC}{CA} = \frac{(35-10) \text{ m s}^{-1}}{(5-0) \text{ s}}$

$= \frac{25 \text{ m s}^{-1}}{5 \text{ s}} = 5 \text{ m s}^{-2}.$

Case (3) : Consider the motion of a body released from a height to fall down vertically, initially from rest with velocity increasing uniformly for 5 s and acquiring the velocity 50 m s^{-1}. Then after hitting the ground, it rises vertically upwards to the same height with velocity decreasing uniformly. In this case, the velocity-time graph is shown in Fig. 2.17 in which part AB shows the downward journey of body with positive velocity while part CD shows the upward journey with negative velocity (since the direction of motion has reversed).

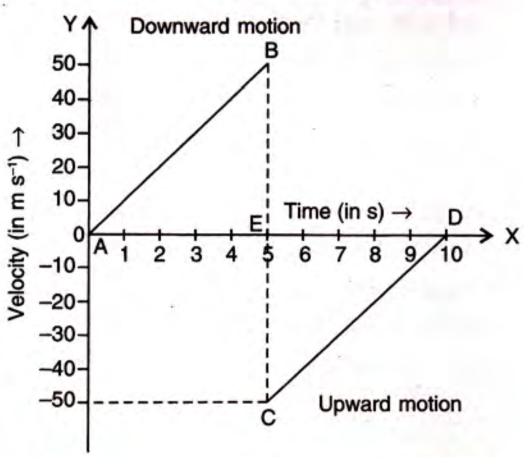


Fig. 2.17 Velocity-time graph for free fall and the rise of a body

(a) In part AB, acceleration = Slope of line AB

$= \frac{50 \text{ m s}^{-1} - 0 \text{ m s}^{-1}}{5 \text{ s}} = 10 \text{ ms}^{-2}$

In part CD, retardation = – Slope of line CD

$= -\frac{0 \text{ m s}^{-1} - 50 \text{ m s}^{-1}}{(10-5) \text{ s}} = 10 \text{ ms}^{-2}$

(b) Total distance travelled in 10 s = area of triangle ABE + area of triangle CDE

$= \frac{1}{2} \times 5 \text{ s} \times 50 \text{ m s}^{-1} + \frac{1}{2} \times 5 \text{ s} \times 50 \text{ m s}^{-1}$

$= 250 \text{ m}$

Displacement in 10 s = 0 (zero).

Since initial and final positions are same.

Conclusions :

(i) (a) For motion with a uniform velocity, the velocity-time graph is a straight line parallel to time axis. (b) If the velocity-time graph is a straight line inclined to the time axis, the motion is with uniform acceleration. (c) If the velocity-time graph is a curve, the motion is with non-uniform acceleration.

(ii) The slope of the straight line (or the tangent to the curve at an instant) gives the acceleration at that instant. (a) The positive slope means velocity increasing with time *i.e.*, accelerated motion. (b) The negative slope means velocity decreasing with time *i.e.*, retardated motion and (c) the zero slope implies motion with constant velocity.

(iii) Knowing the acceleration (*i.e.*, slope) at different instants from the velocity-time graph, we can draw the acceleration-time graph.

(iv) The area enclosed between the velocity-time sketch and the time axis for a certain time interval gives the displacement in that interval of time. The area above the time axis gives the positive displacement, while the area below the time axis gives the negative displacement.

(v) Knowing the distance (or displacement) in different time intervals from the velocity-time graph, we can draw the distance-time (or displacement-time) graph.

2.8 ACCELERATION – TIME GRAPH

In the acceleration-time graph, time is taken on X-axis and acceleration is taken on Y-axis. From this graph, we can find the change in speed in a certain interval of time. For linear motion, acceleration × time = change in speed, therefore *from the area enclosed between the acceleration-time sketch and the time axis, we get the change in speed of the body for the given time interval.*

Let us consider the following cases :

Case (1) : If the body is stationary or if it is moving with a uniform velocity, the acceleration is zero. The acceleration-time graph in such a case is a straight line coinciding with the time axis (Fig 2.18).

Fig. 2.18. Acceleration-time graph for motion with uniform velocity

Case (2) : If the velocity of body in motion increases uniformly with time, the acceleration is constant (*i.e.*, the motion is uniformly accelerated). In such a case, the acceleration-time graph is a straight line parallel to the time axis on the positive acceleration axis. In Fig. 2.19, the straight line *PQ* represents the acceleration-time graph of a body moving with a constant acceleration (= *OP*).

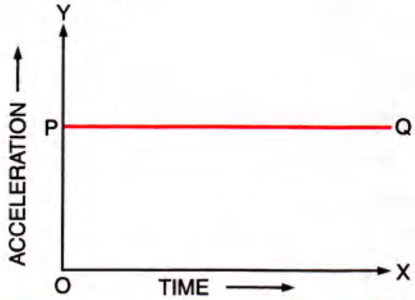


Fig. 2.19. Acceleration-time graph for uniform acceleration

Case (3) : If the velocity of body decreases at a constant rate, the retardation is constant (*i.e.*, the motion is uniformly retarded). The acceleration-time graph is a straight line parallel to the time axis on the negative acceleration axis. In Fig 2.20, the straight line *PQ* represents the acceleration-time graph for a body moving with a constant retardation (= *OP*).

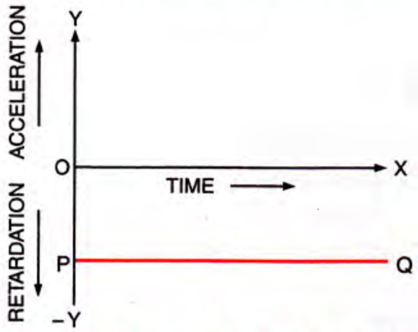


Fig. 2.20. Acceleration-time graph for uniform retardation

Case (4) : If the velocity of body changes in an irregular manner, the acceleration is variable. The acceleration-time graph will then be a curve of any shape.

2.9 MOTION UNDER GRAVITY

A body falling freely under gravity moves with a uniform acceleration of 9·8 m s^{-2} *(or nearly* 10 m s^{-2}*). For a body moving vertically upwards, there is a uniform retardation of* 9·8 m s^{-2}. Thus motion under gravity is an example of uniformly accelerated or uniformly retarded motion.

Here we shall consider the motion of a freely falling body under gravity and we shall use the acceleration-time graph to obtain the velocity-time graph and the displacement-time graph. The acceleration-time graph for such a motion is a straight line parallel to the time axis.

In Fig. 2.21, straight line *AF* represents the acceleration-time graph for a body falling freely (or moving) with uniform acceleration equal to 10 m s^{-2}.

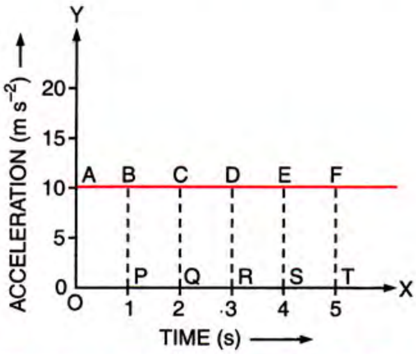


Fig. 2.21 Acceleration-time graph for a freely falling body

This graph can be used to obtain the velocity-time graph, by finding the area enclosed between the straight line and the time axis for each interval of time of 1 s. Let the initial velocity at $t = 0$ be zero, then

Velocity after 1 s = Area *OABP* = $10 \times 1 = 10$ m s^{-1}

Velocity after 2 s = Area *OACQ* = $10 \times 2 = 20$ m s^{-1}

Velocity after 3 s = Area *OADR* = $10 \times 3 = 30$ m s^{-1}

Velocity after 4 s = Area *OAES* = $10 \times 4 = 40$ m s^{-1}

Velocity after 5 s = Area *OAFT* = $10 \times 5 = 50$ m s^{-1}

The velocity-time graph from the above data is shown in Fig. 2.22, which is a straight line OA inclined with the time axis and having a slope of 10 m s^{-2} (which is equal to the acceleration of the body).

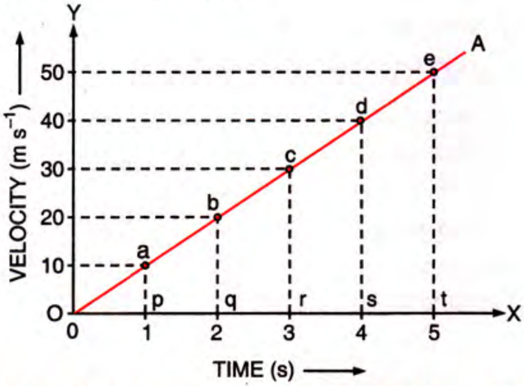


Fig. 2.22 Velocity-time graph for a freely falling body

The velocity-time graph (Fig. 2.22) can be used to obtain the displacement-time graph by finding the area enclosed by the straight line OA with the time axis at each interval of time of 1 s.

Displacement in 1 s = Area of $\Delta\, oap = \frac{1}{2} \times 1 \times 10 = 5$ m

Displacement in 2 s = Area of $\Delta\, obq = \frac{1}{2} \times 2 \times 20 = 20$ m

Displacement in 3 s = Area of $\Delta\, ocr = \frac{1}{2} \times 3 \times 30 = 45$ m

Displacement in 4 s = Area of $\Delta\, ods = \frac{1}{2} \times 4 \times 40 = 80$ m

Displacement in 5 s = Area of $\Delta\, oet = \frac{1}{2} \times 5 \times 50 = 125$ m.

The displacement-time graph from the above data is shown in Fig. 2.23 which is a curve OA (parabola).

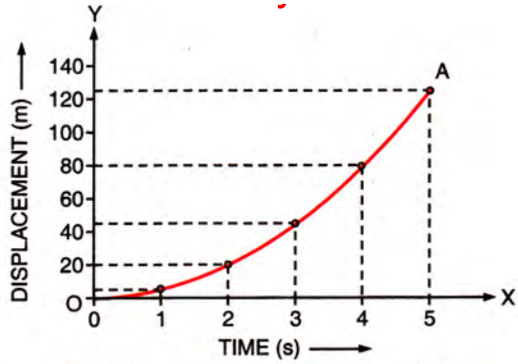


Fig. 2.23 Displacement-time graph for a freely falling body

It may be noted that *for a freely falling body, the displacement is directly proportional to the square of time* ($S \propto t^2$). The table below represents the square of time (t^2) and displacement (S) from the above data.

Square of time t^2 (s^2)	1	4	9	16	25
Displacement S (m)	5	20	45	80	125

Now a graph plotted by taking the displacement (S) on Y-axis and the square of time (t^2) on X-axis is a straight line OA as shown in Fig. 2.24 with

$$\text{the slope} = \frac{(80-20)\text{ m s}^{-1}}{(16-4)\text{ s}} = \frac{60}{12} \text{ m s}^{-2} = 5 \text{ m s}^{-2}.$$

The slope is half the acceleration due to gravity. Thus, the value of acceleration due to gravity (g) can be obtaned by doubling the slope of the $S-t^2$ graph for a freely falling body.

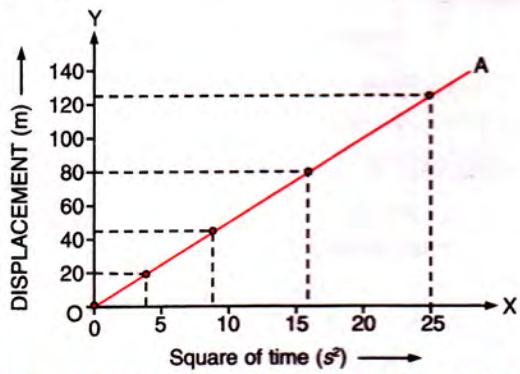


Fig. 2.24 $S - t^2$ graph for a freely falling body

EXAMPLES

1. The following table represents the distance of a car at different instants in a fixed direction.

Time (s)	0	1	2	3	4	5
Distance (m)	0	10	20	30	40	50

(a) Draw displacement–time graph and with its help, find whether the motion of car is uniform or non-uniform?

(b) Use graph to calculate :

(i) the velocity of car

(ii) the displacement of car at $t = 2{\cdot}5$ s and $t = 4{\cdot}5$ s.

(a) The displacement–time graph for the car is shown in Fig. 2.25. Since it is a straight line

OA inclined with the time axis, so the motion of car is with **uniform velocity**.

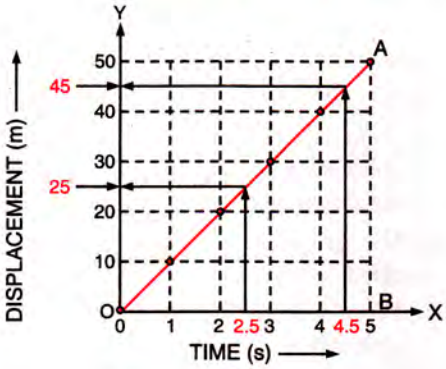


Fig. 2.25

(b) (i) Velocity = Slope of the straight line *OA*

$$= \frac{BA}{OB} = \frac{(50-0)\text{ m}}{(5-0)\text{ s}} = \frac{50\text{ m}}{5\text{ s}} = \mathbf{10\ m\ s^{-1}}$$

(ii) From the graph, it is clear that at t = 2·5 s, displacement is **25 m** and at t = 4·5 s, displacement is **45 m.**

2. Fig 2.26 shows the displacement-time graph for the motion of two boys *A* and *B* along a straight road in the same direction.

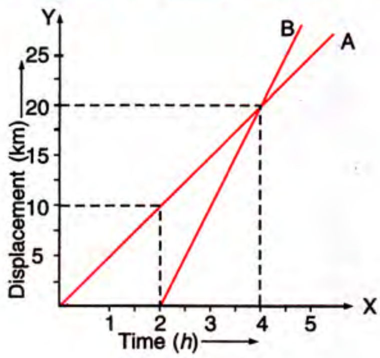


Fig. 2.26

Answer the following :

(i) When did *B* start after *A* ?

(ii) How far away was *A* from *B* when *B* started ?

(iii) Which of the two has greater velocity ?

(iv) When and where did *B* overtake *A* ?

(i) *B* started his motion **2 h later** from the start of *A*.

(ii) When *B* started, *A* was at distance **10 km away from *B*.**

(iii) ***B* has greater velocity than *A*** since the straight line on graph for *B* has greater slope than that for *A*.

(iv) *B* overtook *A* at the instant when both were at the same place. This position is at the point where the two straight lines meet each other. For this point, **distance from the starting point is 20 km and time is 4 h**. Thus *B* overtook *A* when *A* has travelled for 4 h (or *B* has travelled for 4 – 2 = 2 h) at distance 20 km from the starting point

3. A car travels with a uniform velocity of 20 m s^{-1} for 5 s. The brakes are then applied and the car is uniformly retarded. It comes to rest in further 8 s. Draw a graph of velocity against time. Use this graph to find :

(i) the distance travelled in first 5 s,

(ii) the distance travelled after the brakes are applied,

(iii) total distance travelled, and

(iv) acceleration during the first 5 s and last 8 s.

The graph of velocity against time is shown in Fig. 2.27.

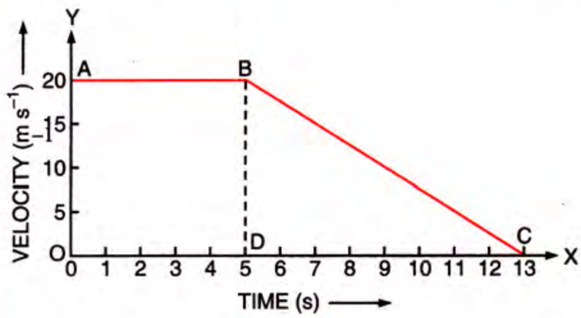


Fig. 2.27

(i) The distance travelled in first 5 s = area of rectangle $OABD = OD \times OA = 5\text{ s} \times 20\text{ m s}^{-1} = \mathbf{100\ m}$

(ii) The distance travelled by car after the brakes are applied = area of $\Delta\, BDC = \frac{1}{2} \times DC \times DB$

$$= \frac{1}{2} \times (13-5)\text{ s} \times 20\text{ m s}^{-1} = \mathbf{80\ m.}$$

(iii) Total distance travelled

= area of rectangle *OABD* + area of triangle *BDC*

= 100 + 80 = **180 m**

(iv) Acceleration in the first 5 s (in part *AB*) = **0**

(since straight line *AB* is parallel to the time axis, so slope = 0).

Acceleration in the last 8 s (in part *BC*)

= Slope of the line *BC*

$$= \frac{BD}{DC} = \frac{(0-20)\text{ m s}^{-1}}{(13-5)\text{ s}} = \frac{-20\text{ m s}}{8\text{ s}}$$

$= \mathbf{-\,2{\cdot}5\ m\ s^{-2}}$

Since acceleration is negative, so retardation $= 2{\cdot}5 \text{ m s}^{-2}$.

4. A train starts from rest and accelerates uniformly at 100 m minute^{-2} for 10 minutes. Find the velocity acquired by the train. It then maintains a constant velocity for 20 minutes. The brakes are then applied and the train is uniformly retarded. It comes to rest in 5 minutes. Draw a velocity–time graph and use it to find :

(i) the retardation in the last 5 minutes,

(ii) total distance travelled, and

(iii) the average velocity of the train.

Initial velocity = 0, time interval = 10 minute, acceleration = 100 m minute^{-2}.

$$\text{Acceleration} = \frac{\text{Final velocity} - \text{Initial velocity}}{\text{Time interval}}$$

$$= \frac{\text{Final velocity} - 0}{\text{Time interval}}$$

or Final velocity = acceleration × time interval

$= 100 \text{ m minute}^{-2} \times 10 \text{ minute}$

$= 1000 \text{ m minute}^{-1}$

∴ The final velocity acquired = 1000 m minute^{-1}.

The velocity–time graph is shown in Fig 2.28.

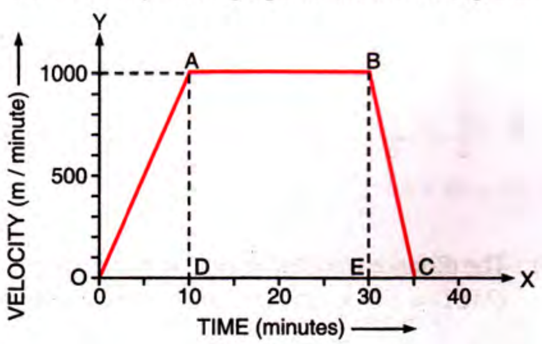


Fig. 2.28

(i) Retardation in the last 5 minutes

$= -$ Slope of the line BC.

$$= -\frac{BE}{EC} = -\frac{(0-1000)\text{ m minute}^{-1}}{(35-30)\text{ minute}}$$

$$= -\frac{-1000 \text{ m minute}^{-1}}{5 \text{ minute}} = \mathbf{200 \text{ m minute}^{-2}}$$

(ii) Total distance travelled

= Area of trapezium $OABC$

$= \frac{1}{2}(OC + AB) \times AD$

$= \frac{1}{2}(35 + 20) \text{ minute} \times 1000 \text{ m minute}^{-1}$

$= 55 \times 500$ m

= **27500 m** (or **27·5 km**).

(iii) Average velocity $= \dfrac{\text{Total distance travelled}}{\text{Total time of travel}}$

$$= \frac{27500 \text{ m}}{35 \text{ minute}} = \mathbf{785{\cdot}7 \text{ m minute}^{-1}}.$$

5. A stone is thrown vertically upwards with an initial velocity of 40 m s^{-1}. Taking g = 10 m s^{-2}, draw the velocity–time graph of the motion of stone till it comes back on the ground.

(i) Use graph to find the maximum height reached by the stone.

(ii) What is the net displacement and total distance covered by the stone ?

(i) Given, $u = 40 \text{ m s}^{-1}$, $g = 10 \text{ m s}^{-2}$.

As the stone rises up, the velocity decreases at the rate of 10 m s^{-2}. When the velocity becomes zero, the stone is at its highest position. Then it begins to fall and its velocity increases at a rate of 10 m s^{-2}. The velocity of stone at different instants is shown in the following table (the upward direction is taken positive).

Time (in s)	0	1	2	3	4	5	6	7	8
Velocity (in m s^{-1})	40	30	20	10	0	–10	–20	–30	–40

Fig. 2.29 shows the velocity–time graph.

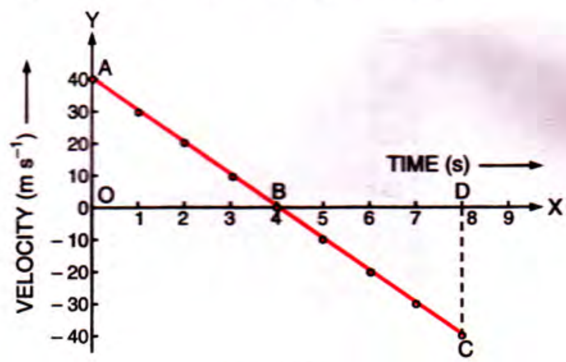


Fig. 2.29

Maximum height reached by the stone

= Area of Δ OAB

$= \frac{1}{2} OB \times OA$

$= \frac{1}{2} \times 4 \text{ s} \times 40 \text{ m s}^{-1} = \mathbf{80 \text{ m}}.$

(ii) Net displacement

= Area of Δ OAB – Area of Δ BDC

$= \frac{1}{2} OB \times OA - \frac{1}{2} BD \times DC$

$= (\frac{1}{2} \times 4 \text{ s} \times 40 \text{ m s}^{-1}) - (\frac{1}{2} \times 4 \text{ s} \times 40 \text{ m s}^{-1}) = \mathbf{0}$

Total distance covered

$= \text{Area of } \Delta\, OAB + \text{Area of } \Delta\, BDC$
$= 80 \text{ m} + 80 \text{ m} = \mathbf{160 \text{ m}}$

6. A car starting from rest, accelerates at a rate of 2 m s^{-2} for 5 s. For this journey, (a) draw the velocity-time graph (b) draw the displacement-time graph using the velocity-time graph in part (a).

(a) Given, $u = 0$, $a = 2 \text{ m s}^{-2}$.

The velocity of car at different instants is given in the table below :

Time (in s)	0	1	2	3	4	5
Velocity (in m s^{-1})	0	2	4	6	8	10

Fig. 2.30 shows the velocity-time graph.

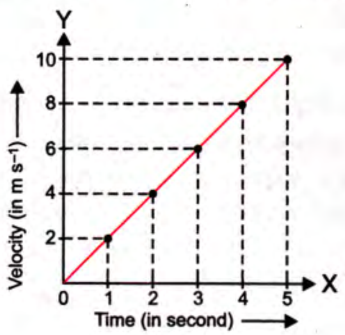


Fig. 2.30 Velocity-time graph

(b) From Fig. 2.30, the displacement of car at any instant can be obtained by finding the area enclosed by the straight line with the time axis up to that instant.

At $t = 1$ s, displacement $S = \frac{1}{2} \times 1 \times 2 = 1 \text{ m}$

At $t = 2$ s, displacement $S = \frac{1}{2} \times 2 \times 4 = 4 \text{ m}$

At $t = 3$ s, displacement $S = \frac{1}{2} \times 3 \times 6 = 9 \text{ m}$

At $t = 4$ s, displacement $S = \frac{1}{2} \times 4 \times 8 = 16 \text{ m}$

At $t = 5$ s, displacement $S = \frac{1}{2} \times 5 \times 10 = 25 \text{ m}$

The table below gives the displacement of car at different instants.

Time (in s)	0	1	2	3	4	5
Displacement (in m)	0	1	4	9	16	25

The displacement-time graph is shown in Fig. 2.31.

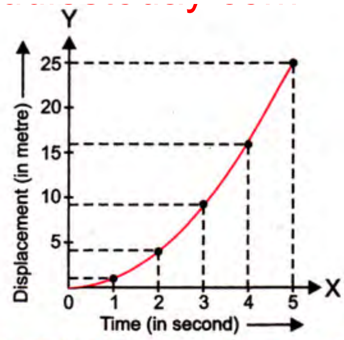


Fig. 2.31 Displacement-time graph

7. The following table represents the velocity of a moving body at different instants of time.

Time (s)	0	5	10	15	20	25	30
Velocity (m s^{-1})	10	15	20	20	30	15	0

Draw the velocity–time graph and answer the following :

(i) For which interval of time the body has a uniform motion ? Find the velocity in this time interval?

(ii) For which interval of time the body has the accelerated motion ? Calculate the acceleration.

(iii) For which interval of time, the body has retardation ? Calculate the retardation.

The velocity–time graph is shown in Fig. 2.32.

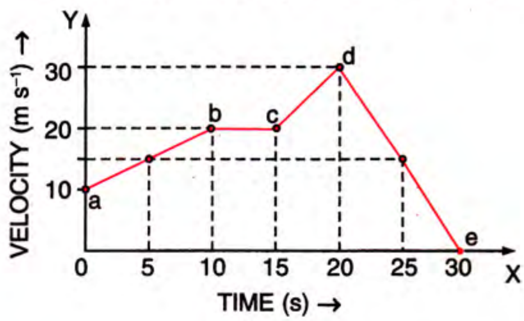


Fig. 2.32

(i) The body has uniform motion from t **= 10 s to** t **= 15 s** in part *bc* since velocity is constant and is equal to 20 m s^{-1} during this interval.

(ii) The body has the accelerated motion from t **= 0 s to** t **= 10 s** in part *ab*, and also from t **= 15 s to** t **= 20 s** in part *cd* since velocity is increasing with time during these intervals.

From $t = 0$ to $t = 10$ s, in part *ab*

Acceleration a = Slope of the straight line *ab*

$$= \frac{(20-10)\text{ m s}^{-1}}{(10-0)\text{ s}} = \frac{10 \text{ m s}^{-1}}{10 \text{ s}} = \mathbf{1 \text{ m s}^{-2}}$$

From $t = 15$ s to $t = 20$ s, in part *cd*

Acceleration a' = Slope of the line *cd*

$$= \frac{(30-20)\,\text{m s}^{-1}}{(20-15)\,\text{s}} = \frac{10\,\text{m s}^{-1}}{5\,\text{s}} = \mathbf{2\ m\ s^{-2}}$$

(iii) The body has retardation **from $t = 20$ s to $t = 30$ s**, in part *de*.

Retardation = – Slope of the line *de*

$$= -\frac{(0-30)\,\text{m s}^{-1}}{(30-20)\,\text{s}} = -\frac{-30\,\text{m s}^{-1}}{10\,\text{s}} = \mathbf{3\ m\ s^{-2}}$$

EXERCISE 2 (B)

1. For the motion with uniform velocity, how is the distance travelled related to the time ?

   Ans. Distance is directly proportional to time.

2. What informations about the motion of a body are obtained from the displacement–time graph ?

3. (a) What does the slope of a displacement–time graph represent ?

   (b) Can displacement–time sketch be parallel to the displacement axis ? Give reason to your answer.

4. What can you say about the nature of motion of a body if its displacement–time graph is

   (a) a straight line parallel to time axis ?

   (b) a straight line inclined to the time axis with an acute angle ?

   (c) a straight line inclined to the time axis with an obtuse angle ?

   (d) a curve.

   Ans. (a) body is stationary (or no motion), (b) motion away from the starting point with uniform velocity (c) motion towards the starting point with uniform velocity (d) motion with variable velocity.

5. Draw a displacement–time graph for a boy going to school with a uniform velocity.

6. State how the velocity–time graph can be used to find (i) the acceleration of a body, (ii) the distance travelled by the body in a given time, and (iii) the displacement of the body in a given time.

7. Fig. 2.33 shows displacement–time graph of two vehicles *A* and *B* moving along a straight road. Which vehicle is moving faster ? Give reason.

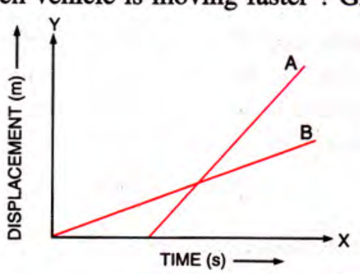


Fig. 2.33 **Ans.** Vehicle *A*

Reason : Slope of line *A* is more than that of line *B*.

8. State the type of motion represented by the following sketches in Fig. 2.34 (a) and (b).

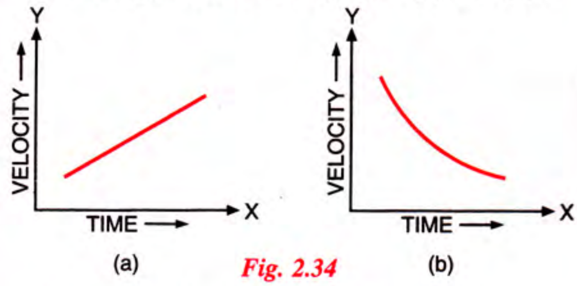


Fig. 2.34

Give example of each type of motion.

Ans. (a) Uniformly accelerated motion e.g. motion of a body released downward.

(b) Motion with a variable retardation e.g. a car approaching its destination.

9. Draw a velocity–time graph for a body moving with an initial velocity u and uniform acceleration a. Use this graph to find the distance travelled by the body in time t.

10. What does the slope of velocity–time graph represent ? **Ans.** Acceleration

11. Fig 2.35 shows the velocity–time graph for two cars *A* and *B* moving in same direction. Which car has the greater acceleration ? Give reason to your answer.

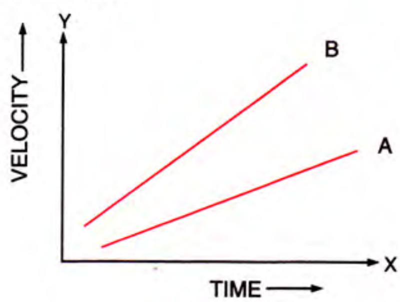


Fig. 2.35 **Ans.** *B*

Reason : Slope of straight line for car *B* is more than that of line *A*.

12. Fig. 2.36 shows the displacement–time graph for four bodies *A*, *B*, *C* and *D*. In each case state what information do you get about the acceleration (zero, positive or negative).

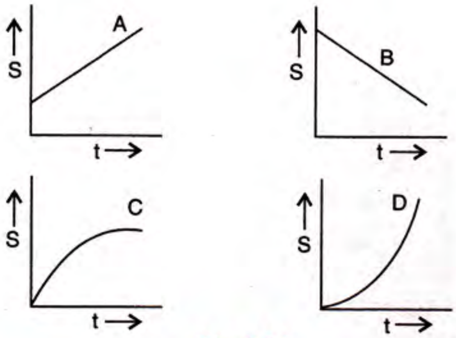


Fig. 2.36

Ans. ***A*** : Zero acceleration since slope (*i.e.*, velocity) is constant, ***B*** : zero acceleration since slope is constant, ***C*** : negative acceleration (or retardation) since slope is decreasing with time, ***D*** : positive acceleration since slope is increasing with time.

13. Draw the shape of the velocity–time graph for a body moving with (a) uniform velocity, (b) uniform acceleration.

14. The velocity-time graph for a uniformly retarded body is a straight line inclined to the time axis with an obtuse angle. How is retardation calculated from the velocity-time graph ?

Ans. By finding the negative slope.

15. Draw a graph for acceleration against time for a uniformly accelerated motion. How can it be used to find the change in speed in a certain interval of time ?

16. Draw a velocity–time graph for the free fall of a body under gravity, starting from rest. Take $g = 10$ m s^{-2}.

17. How is the distance related with time for the motion under uniform acceleration such as the motion of a freely falling body ? **Ans.** $S \propto t^2$

18. A body falls freely from a certain height. Show graphically the relation between the distance fallen and square of time. How will you determine g from this graph ?

Multiple choice type :

1. The velocity-time graph of a body in motion is a straight line inclined to the time axis. The correct statement is :
   (a) velocity is uniform
   (b) acceleration is uniform
   (c) both velocity and acceleration are uniform
   (d) neither velocity nor acceleration is uniform.

   Ans. (b) acceleration is uniform.

2. For uniform motion :
   (a) the distance-time graph is a straight line parallel to the time axis.
   (b) the speed-time graph is a straight line inclined to the time axis.
   (c) the speed-time graph is a straight line parallel to the time axis.
   (d) the acceleration-time graph is a straight line parallel to the time axis.

   Ans. (c) the speed-time graph is a straight line parallel to the time axis.

3. For a uniformly retarded motion, the velocity-time graph is :
   (a) a curve
   (b) a straight line parallel to the time axis
   (c) a straight line perpendicular to the time axis.
   (d) a straight line inclined to the time axis.

   Ans. (d) a straight line inclined to the time axis

Numericals :

1. Fig. 2.37 (a) shows the displacement–time graph for the motion of a body. Use it to calculate the velocity of body at $t = 1$ s, 2 s and 3 s, then draw the velocity–time graph for it in Fig. (b).

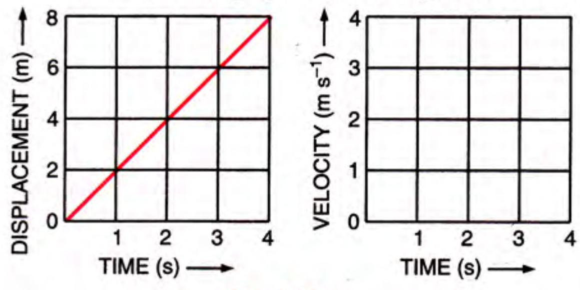


Fig. 2.37

2. Following table gives the displacement of a car at different instants of time.

Time (s)	0	1	2	3	4
Displacement (m)	0	5	10	15	20

   (a) Draw the displacement–time sketch and find the average velocity of car.
   (b) What will be the displacement of car at (i) 2·5 s and (ii) 4·5 s ?

   Ans. (a) 5 m s^{-1}, (b) (i) 12·5 m, (ii) 22·5 m.

3. A body is moving in a straight line and its displacement at various instants of time is given in the following table :

Time (s)	0	1	2	3	4	5	6	7
Displacement (m)	2	6	12	12	12	18	22	24

   Plot displacement–time graph and calculate :
   (i) total distance travelled in interval 1 s to 5 s,
   (ii) average velocity in time interval 1 s to 5 s.

   Ans. (i) 12 m (ii) 3 m s^{-1}

4. Fig. 2.38 shows the displacement of a body at different times.

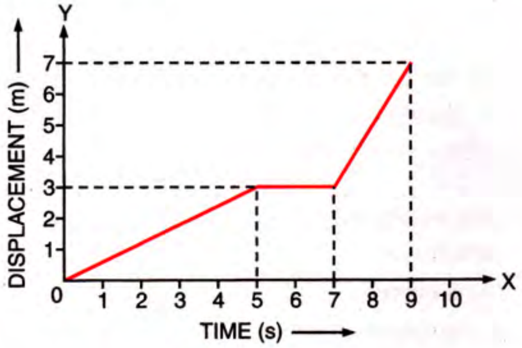


Fig. 2.38

(a) Calculate the velocity of the body as it moves for time interval (i) 0 to 5 s, (ii) 5 s to 7 s and (iii) 7 s to 9 s.

(b) Calculate the average velocity during the time interval 5 s to 9 s.

[**Hint** : From 5 s to 9 s, displacement = 7 m − 3 m = 4 m]

Ans. (a) (i) $0{\cdot}6$ m s^{-1}, (ii) 0 m s^{-1}, (iii) 2 m s^{-1}, (b) 1 m s^{-1}

5. From the displacement–time graph of a cyclist, given in Fig. 2.39, find :

(i) the average velocity in the first 4 s,

(ii) the displacement from the initial position at the end of 10 s,

(iii) the time after which he reaches the starting point.

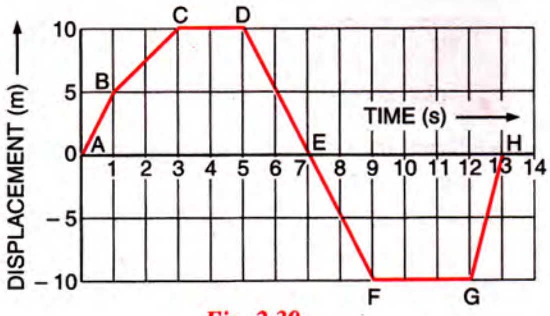


Fig. 2.39

Ans. (i) $2{\cdot}5$ m s^{-1}, (ii) −10 m, (iii) 7 s and 13 s.

6. Fig. 2.40 ahead represents the displacement–time sketch of motion of two cars *A* and *B*. Find :

(i) the distance by which the car *B* was initially ahead of car *A*.

(ii) the velocities of car *A* and car *B*.

(iii) the time in which the car *A* catches the car *B*.

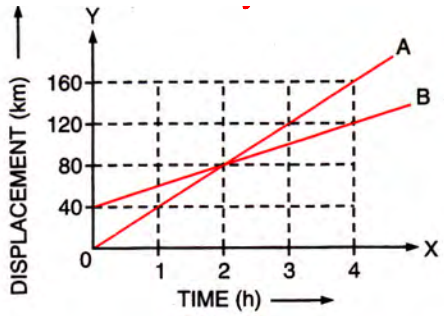


Fig. 2.40

(iv) the distance from start when the car *A* will catch the car *B*.

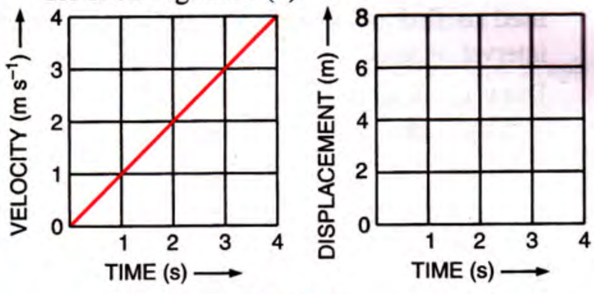
Ans. (i) 40 km, (ii) ***A***— 40 km h^{-1}, ***B***— 20 km h^{-1} (iii) 2 h, (iv) 80 km.

7. A body at rest is made to fall from the top of a tower. Its displacement at different instants is given in the following table :

Time (in s)	0·1	0·2	0·3	0·4	0·5	0·6
Displacement (in m)	0·05	0·20	0·45	0·80	1·25	1·80

Draw a displacement–time graph and state whether the motion is uniform or non-uniform ?

8. Fig. 2.41 (a) shows the velocity–time graph for the motion of a body. Use it to find the displacement of the body at t = 1 s, 2 s, 3 s and 4 s, then draw the displacement–time graph for it on Fig. 2.41 (b).



(a) *Fig. 2.41* (b)

9. Fig. 2.42 given below shows a velocity–time graph for a car starting from rest. The graph has three parts *AB*, *BC* and *CD*.

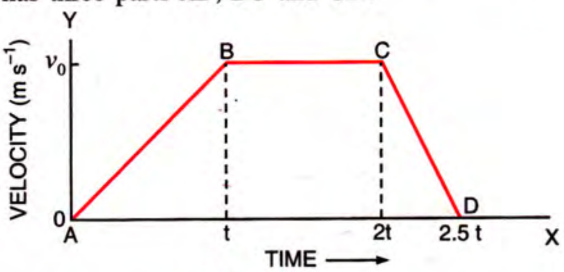


Fig. 2.42

(i) State how is the distance travelled in any part determined from this graph.

(ii) Compare the distance travelled in part *BC* with the distance travelled in part *AB*.

(iii) Which part of graph shows motion with uniform (a) velocity (b) acceleration (c) retardation ?

(iv) (a) Is the magnitude of acceleration higher or lower than that of retardation ? Give a reason. (b) Compare the magnitude of acceleration and retardation.

Ans. (i) By finding the area enclosed by the graph in that part with the time axis (ii) 2 : 1 (iii) (a) *BC* (b) *AB* (c) *CD* (iv) (a) lower, as slope of line *AB* is less than that of the line *CD*, (b) 1 : 2.

10. The velocity–time graph of a moving body is given below in Fig. 2.43.

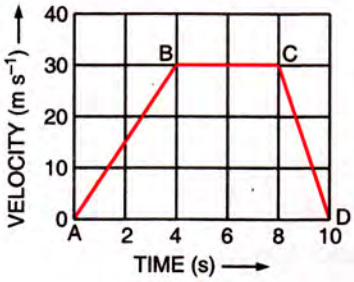


Fig. 2.43

Find :

(i) the acceleration in parts *AB*, *BC* and *CD*.

(ii) displacement in each part *AB*, *BC*, *CD*, and

(iii) total displacement.

Ans. (i) ***AB*** : 7·5 m s^{-2}, ***BC*** : 0 m s^{-2}, ***CD*** : – 15 m s^{-2} (ii) ***AB*** : 60 m, ***BC*** : 120 m, ***CD*** : 30 m (iii) 210 m

11. A ball moves on a smooth floor in a straight line with a uniform velocity 10 m s^{-1} for 6 s. At $t = 6$ s, the ball hits a wall and comes back along the same line to the starting point with same speed. Draw the velocity-time graph and use it to find the total distance travelled by the ball and its displacement.

Ans. Distance = 120 m, displacement = 0.

12. Fig. 2.44 shows the velocity-time graph of a particle moving in a straight line.

(i) State the nature of motion of particle.

(ii) Find the displacement of particle at $t = 6$ s.

(iii) Does the particle change its direction of motion ?

(iv) Compare the distance travelled by the particle from 0 to 4 s and from 4 s to 6 s.

(v) Find the acceleration from 0 to 4 s and retardation from 4 s to 6 s.

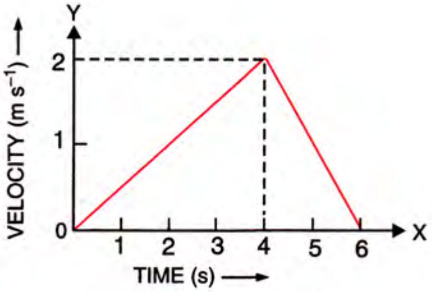


Fig. 2.44

Ans. (i) Uniformly accelerated from 0 to 4 s and then uniformly retarded from 4 s to 6 s. (ii) 6 m (iii) No, (iv) 2 : 1 (v) acceleration = 0·5 m s^{-2}, retardation = 1 m s^{-2}.

(C) EQUATIONS OF MOTION

2.9 EQUATIONS OF UNIFORMLY ACCELERATED MOTION

For motion of a body moving with a uniform acceleration, the following *three* equations give the relationship between initial velocity (u), final velocity (v), acceleration (a), time of journey (t) and distance travelled (S) :

$$\left.\begin{aligned}&(1)\ v = u + at\\ &(2)\ S = \frac{1}{2}(u+v)\,t = ut + \frac{1}{2}at^2 \text{ and}\\ &(3)\ v^2 = u^2 + 2aS\end{aligned}\right\}\quad ...(2.8)$$

Derivation

(1) Graphical method (from velocity-time graph)

Consider the linear motion of a body with an initial velocity u. The body accelerates uniformly and in time t, it acquires the final velocity v. The velocity-time graph is a straight line *AB* as shown in Fig. 2.45.

It is evident from the graph that

Initial velocity (at $t = 0$) = $OA = u$

Final velocity (at time t) = $OC = v$

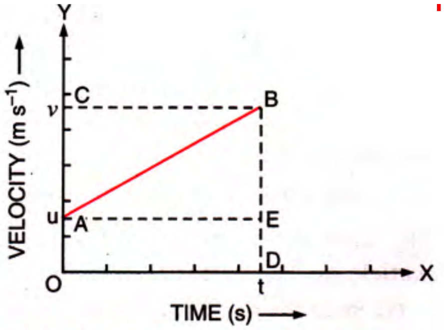


Fig. 2.45 Graph showing the linear motion with uniform acceleration

(i) Acceleration $a =$ Slope of the line AB

or $\quad a = \dfrac{EB}{AE} = \dfrac{AC}{OD} = \dfrac{OC - OA}{OD} = \dfrac{v-u}{t}$

or $\quad a\,t = v - u$

or $\quad \boxed{v = u + at}$ ... (2.9)

(ii) The distance S travelled in time t

= area of the trapezium $OABD$

= area of rectangle $OAED$ + area of triangle ABE

or $S = OA \times OD + \dfrac{1}{2} \times BE \times AE$

$= u \times t + \dfrac{1}{2} \times (v - u) \times t$ ..(2.10)

But from eqn (2.9), $v - u = at$

$\therefore$ From eqn. (2.10),

$$\boxed{S = ut + \frac{1}{2}at^2} \quad ...(2.11)$$

(iii) The distance S travelled in time t

= area of the trapezium $OABD$

or $\quad S = \dfrac{1}{2}(OA + DB) \times OD$

or $\quad S = \dfrac{1}{2}(u + v) \times t$ ... (2.12)

From eqn. (2.9), $t = \dfrac{v-u}{a}$

$\therefore$ From eqn (2.12),

$\therefore \quad S = \dfrac{1}{2}(u+v) \times \left(\dfrac{v-u}{a}\right) = \dfrac{1}{2}\left(\dfrac{v^2-u^2}{a}\right)$

or $\quad 2\,a\,S = v^2 - u^2$

or $\quad \boxed{v^2 = u^2 + 2aS}$ ... (2.13)

(2) Alternative method

(i) By definition,

$$\text{Acceleration} = \frac{\text{Change in velocity}}{\text{Time taken}}$$

$$= \frac{\text{Final velocity} - \text{Initial velocity}}{\text{Time taken}}$$

or $\quad a = \dfrac{v-u}{t}$

or $\quad at = v - u$

or $\quad v = u + at$ ...(1)

(ii) Distance travelled = Average velocity × time

$$= \left(\frac{\text{Initial velocity} + \text{Final velocity}}{2}\right) \times \text{time}$$

or $\quad S = \dfrac{u+v}{2} \times t$

But from eqn. (1), $v = u + at$

$\therefore \quad S = \dfrac{u + (u + at)}{2} \times t$

or $\quad S = \left(\dfrac{2u + at}{2}\right) \times t$

or $\quad S = ut + \dfrac{1}{2}at^2$ ...(2)

(iii) Distance travelled = Average velocity × time

or $\quad S = \dfrac{u+v}{2} \times t$

But from eqn. (1), $v = u + at$

or $\quad t = \dfrac{v-u}{a}$

$\therefore \quad S = \dfrac{u+v}{2} \times \dfrac{v-u}{a} = \dfrac{v^2-u^2}{2a}$

or $\quad v^2 - u^2 = 2aS$

or $\quad v^2 = u^2 + 2aS$ ...(3)

Equations (1), (2) and (3) are same as the equations (2.9), (2.11) and (2.13).

Special cases

(a) When a body starts from rest, initial velocity is zero ($u = 0$), then

$$\left.\begin{aligned} &\text{(i)} & v &= at \\ &\text{(ii)} & S &= \frac{1}{2}at^2 \\ &\text{(iii)} & v^2 &= 2aS \end{aligned}\right\} \quad ...(2.14)$$

(b) If a body is moving with a uniform retardation, a will be negative. The equations of motion then take the form :

$$\left.\begin{aligned} &\text{(i)} & v &= u - at \\ &\text{(ii)} & S &= ut - \frac{1}{2}at^2 \\ &\text{(iii)} & v^2 &= u^2 - 2aS \end{aligned}\right\} \quad ...(2.15)$$

1. A car acquires a velocity of 72 km h^{-1} in 10 s starting from rest. Calculate :
(i) the acceleration,
(ii) the average velocity, and
(iii) the distance travelled in this time.

Given, initial velocity $u = 0$

Final velocity $v = 72$ km h^{-1}

$$= \frac{72 \times 1000 \text{ m}}{60 \times 60 \text{ s}} = 20 \text{ m s}^{-1}$$

Time taken $t = 10$ s

(i) Acceleration $a = \dfrac{v-u}{t}$

$$= \frac{(20-0) \text{ m s}^{-1}}{10 \text{ s}} = \mathbf{2 \text{ m s}^{-2}}$$

(ii) Average velocity $= \dfrac{u+v}{2}$

$$= \frac{0+20}{2} \text{ m s}^{-1} = \mathbf{10 \text{ m s}^{-1}}$$

(iii) Distance travelled S = average velocity × time
$= (10 \text{ m s}^{-1}) \times (10 \text{ s})$
$= \mathbf{100 \text{ m}}$

Alternative method :

Distance travelled $S = ut + \frac{1}{2} at^2$

$$= 0 + \frac{1}{2} \times 2 \times (10)^2$$

$= \mathbf{100 \text{ m}}$

2. A ball is initially moving with a velocity 0·5 m s^{-1}. Its velocity decreases at a rate of 0·05 m s^{-2}. (a) How much time will it take to stop ? (b) How much distance will the ball travel before it stops ?

Given, initial velocity $u = 0{\cdot}5$ m s^{-1}, final velocity $v = 0$, acceleration $a = -0{\cdot}05$ m s^{-2} (Here negative sign is used since velocity decreases with time).

(a) From equation of motion $v = u + at$

$$0 = 0{\cdot}5 - 0{\cdot}05 \times t$$

or $0{\cdot}05\, t = 0{\cdot}5$

or $t = \dfrac{0{\cdot}5}{0{\cdot}05} = \mathbf{10 \text{ s}}$

(b) From equation of motion $v^2 = u^2 + 2a\,S$

$$0 = (0{\cdot}5)^2 - 2 \times 0{\cdot}05 \times S$$

or $0{\cdot}1\, S = 0{\cdot}25$ or $S = \dfrac{0{\cdot}25}{0{\cdot}1} = \mathbf{2{\cdot}5 \text{ m}}$

3. A body initially at rest travels a distance 100 m in 5 s with a constant acceleration. Calculate : (i) the acceleration, and (ii) the final velocity at the end of 5 s.

Given, initial velocity $u = 0$, distance $S = 100$ m, time taken $t = 5$ s.

(i) From equation of motion $S = ut + \frac{1}{2} at^2$

$$100 = 0 \times 5 + \frac{1}{2} \times a \times (5)^2$$

or $100 = \frac{1}{2} \times 25\, a$

or Acceleration $a = \dfrac{100 \times 2}{25} = \mathbf{8 \text{ m s}^{-2}}$

(ii) From equation of motion $v = u + at$

Final velocity $v = 0 + 8 \times 5 = \mathbf{40 \text{ m s}^{-1}}$.

4. A car initially at rest starts moving with a constant acceleration of 0·5 m s^{-2} and travels a distance of 25 m. Find : (i) its final velocity and (ii) the time taken.

Given, initial velocity $u = 0$,

acceleration $a = 0{\cdot}5$ m s^{-2}

distance travelled $S = 25$ m.

(i) From equation of motion $v^2 = u^2 + 2aS$

$$v^2 = (0)^2 + 2 \times 0{\cdot}5 \times 25$$

or $v^2 = 25$

or Final velocity $v = \sqrt{25} = \mathbf{5 \text{ m s}^{-1}}$

(ii) From equation of motion $v = u + at$

$5 = 0 + 0{\cdot}5 \times t$ or $0{\cdot}5\, t = 5$

$\therefore$ Time taken $t = \dfrac{5}{0{\cdot}5} = \mathbf{10 \text{ s.}}$

5. A body moving with uniform acceleration travels 84 m in the first 6 s and 180 m in the next 5 s. Find : (a) the initial velocity, and (b) the acceleration of the body.

Let u be the initial velocity and a be the acceleration of the body.

Given, $S_1 = 84$ m, $t_1 = 6$ s,

$S_2 = 84 + 180 = 264$ m and $t_2 = 6 + 5 = 11$ s

From relation $S = ut + \frac{1}{2} at^2$

Distance travelled in 6 s,

$$84 = u \times 6 + \frac{1}{2} a \times (6)^2$$

or $6u + 18a = 84$ or $u + 3a = 14$ .... (i)

Distance travelled in 11 s,

$$264 = u \times 11 + \frac{1}{2} a \times (11)^2$$

or $11u + \frac{121}{2}a = 264$ or $u + \frac{11}{2}a = 24$ .... (ii)

On solving eqns. (i) and (ii),

Initial velocity of body $u = \mathbf{2\ m\ s^{-1}}$

and acceleration $a = \mathbf{4\ m\ s^{-2}}$.

6. A body with an initial velocity of 18 km h⁻¹ accelerates uniformly at the rate of 9 cm s⁻² over a distance of 200 m. Calculate :

(i) the acceleration in m s⁻².

(ii) its final velocity in m s⁻¹.

(i) Acceleration $= 9\ \text{cm s}^{-2} = \frac{9}{100}\ \text{m s}^{-2}$

$= \mathbf{0{\cdot}09\ m\ s^{-2}}$

(ii) Given, initial velocity $u = 18\ \text{km h}^{-1}$

$= \frac{18000\ \text{m}}{60 \times 60\ \text{s}} = 5\ \text{m s}^{-1}$

Acceleration $a = 0{\cdot}09\ \text{m s}^{-2}$ and distance $S = 200$ m

From equation of motion $v^2 = u^2 + 2aS$

$v^2 = (5)^2 + 2 \times 0{\cdot}09 \times 200$

or $v^2 = 25 + 36 = 61$

∴ Final velocity $v = \sqrt{61} = \mathbf{7{\cdot}81\ m\ s^{-1}}$.

7. A particle initially at rest, moves with an acceleration 5 m s⁻² for 5 s. Find the distance travelled in (a) 4 s, (b) 5 s and (c) 5th second.

Given, initial velocity $u = 0$,

acceleration $a = 5\ \text{m s}^{-2}$

(a) Distance travelled in $t = 4$ s,

$S_1 = ut + \frac{1}{2}at^2 = 0 \times 4 + \frac{1}{2} \times 5 \times (4)^2$

$= \mathbf{40\ m}$

(b) Distance travelled in $t = 5$ s,

$S_2 = ut + \frac{1}{2}at^2 = 0 \times 5 + \frac{1}{2} \times 5 \times (5)^2$

$= \mathbf{62{\cdot}5\ m}$

(c) Distance travelled in 5th second,

= Distance travelled in 5 s – Distance travelled in 4 s

$= S_2 - S_1 = (62{\cdot}5 - 40)\ \text{m} = \mathbf{22{\cdot}5\ m}$

8. A particle starts to move in a straight line from a point with velocity 10 m s⁻¹ and acceleration – 2·0 m s⁻². Find the position and velocity of the particle at (i) *t* = 5 s, (ii) *t* = 10 s.

Given, $u = 10\ \text{m s}^{-1}$, $a = -2{\cdot}0\ \text{m s}^{-2}$

(i) Displacement at $t = 5$ s is

$$S = ut + \frac{1}{2}at^2$$
$$= 10 \times 5 + \frac{1}{2} \times (-2{\cdot}0) \times (5)^2$$
$$= 50 - 25 = 25\ \text{m}$$

i.e., after 5 s, the particle will be at distance **25 m** from the starting point.

Velocity at $t = 5$ s is

$$v = u + at$$

or $v = 10 + (-2{\cdot}0) \times 5 = \mathbf{0}$

i.e., the particle is momentarily at rest at $t = 5$ s.

(ii) Displacement at $t' = 10$ s is

$$S' = ut' + \frac{1}{2}at'^2$$
$$= 10 \times 10 + \frac{1}{2} \times (-2{\cdot}0) \times (10)^2$$
$$= 100 - 100 = 0\ \text{(zero)}$$

i.e., after 10 s, the particle has **come back to the starting point.**

Velocity at $t' = 10$ s is

$$v = u + at'$$

or $v = 10 + (-2{\cdot}0) \times 10$

$= \mathbf{-10\ m\ s^{-1}}$.

i.e., velocity is 10 m s⁻¹ towards the starting point (*i.e.*, opposite to the initial direction of motion).

EXERCISE 2(C)

1. Write *three* equations of uniformly accelerated motion relating the initial velocity (u), final velocity (v), time (t), acceleration (a) and displacement (S).
2. Derive following equations for a uniformly accelerated motion :

   (i) $v = u + at$

   (ii) $S = ut + \frac{1}{2}at^2$

   (iii) $v^2 = u^2 + 2aS$

   where the symbols have their usual meanings.
3. Write an expression for the distance S covered in time t by a body which is initially at

rest and starts moving with a constant acceleration a.

Ans. $S = \frac{1}{2} at^2$

Multiple choice type :

1. The correct equation of motion is :
   (a) $v = u + aS$ (b) $v = ut + a$
   (c) $S = ut + \frac{1}{2} at$ (d) $v = u + at$
   Ans. (d) $v = u + at$

2. A car starting from rest accelerates uniformly to acquire a speed 20 km h^{-1} in 30 min. The distance travelled by car in this time interval will be :
   (a) 600 km (b) 5 km
   (c) 6 km (d) 10 km **Ans.** (b) 5 km

Numericals :

1. A body starts from rest with a uniform acceleration 2 m s^{-2}. Find the distance covered by the body in 2 s. **Ans.** 4 m

2. A body starts with an initial velocity of 10 m s^{-1} and acceleration 5 m s^{-2}. Find the distance covered by it in 5 s. **Ans.** 112·5 m

3. A vehicle is accelerating on a straight road. Its velocity at any instant is 30 km h^{-1}, after 2 s, it is 33·6 km h^{-1} and after further 2 s, it is 37·2 km h^{-1}. Find the acceleration of vehicle in m s^{-2}. Is the acceleration uniform ?
   Ans. 0·5 m s^{-2}, Yes

4. A body, initially at rest, starts moving with a constant acceleration 2 m s^{-2}. Calculate : (i) the velocity acquired and (ii) the distance travelled in 5 s. **Ans.** (i) 10 m s^{-1}, (ii) 25 m

5. A bullet initially moving with a velocity 20 m s^{-1} strikes a target and comes to rest after penetrating a distance 10 cm in the target. Calculate the retardation caused by the target.
   Ans. 2000 m s^{-2}

6. A train moving with a velocity of 20 m s^{-1} is brought to rest by applying brakes in 5 s. Calculate the retardation. **Ans.** 4 m s^{-2}

7. A train travels with a speed of 60 km h^{-1} from station A to station B and then comes back with a speed 80 km h^{-1} from station B to station A. Find : (i) the average speed, and (ii) the average velocity of train.
   Ans. (i) 68·57 km h^{-1}, (ii) zero

8. A train is moving with a velocity of 90 km h^{-1}. It is brought to stop by applying the brakes which produce a retardation of 0·5 m s^{-2}. Find : (i) the velocity after 10 s, and (ii) the time taken by the train to come to rest.
   Ans. (i) 20 m s^{-1}, (ii) 50 s

9. A car travels a distance 100 m with a constant acceleration and average velocity of 20 m s^{-1}. The final velocity acquired by the car is 25 m s^{-1}. Find : (i) the initial velocity and (ii) acceleration of car. **Ans.** (i) 15 m s^{-1} (ii) 2 m s^{-2}

10. When brakes are applied to a bus, the retardation produced is 25 cm s^{-2} and the bus takes 20 s to stop. Calculate : (i) the initial velocity of bus, and (ii) the distance travelled by bus during this time. **Ans.** (i) 5 m s^{-1}, (ii) 50 m

11. A body moves from rest with a uniform acceleration and travels 270 m in 3 s. Find the velocity of the body at 10 s after the start.
    Ans. 600 m s^{-1}

12. A body moving with a constant acceleration travels the distances 3 m and 8 m respectively in 1 s and 2 s. Calculate : (i) the initial velocity, and (ii) the acceleration of body.
    Ans. (i) 2 m s^{-1}, (ii) 2 m s^{-2}

13. A car travels with a uniform velocity of 25 m s^{-1} for 5 s. The brakes are then applied and the car is uniformly retarded and comes to rest in further 10 s. Find : (i) the distance which the car travels before the brakes are applied, (ii) the retardation, and (iii) the distance travelled by the car after applying the brakes.
    Ans. (i) 125 m, (ii) 2·5 m s^{-2}, (iii) 125 m

14. A space craft flying in a straight course with a velocity of 75 km s^{-1} fires its rocket motors for 6·0 s. At the end of this time, its speed is 120 km s^{-1} in the same direction. Find : (i) the space craft's average acceleration while the motors were firing, (ii) the distance travelled by the space craft in the first 10 s after the rocket motors were started, the motors having been in action for only 6·0 s.
    Ans. (i) 7·5 km s^{-2}, (ii) 1065 km

15. A train starts from rest and accelerates uniformly at a rate of 2 m s^{-2} for 10 s. It then maintains a constant speed for 200 s. The brakes are then applied and the train is uniformly retarded and comes to rest in 50 s. Find : (i) the maximum velocity reached, (ii) the retardation in the last 50 s, (iii) the total distance travelled, and (iv) the average velocity of the train.
    Ans. (i) 20 m s^{-1}, (ii) 0·4 m s^{-2}, (iii) 4600 m, (iv) 17·69 m s^{-1}